Advancing assessment in fractions: designing and implementing formative proficiency tasks

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While many studies have focused on the challenges students face in the domain of fractions and the corresponding pedagogies of teaching and learning, there has been inadequate and disproportionate attention dedicated to assessment resources, particularly those tailored for formative assessment in the context of fractions. This study takes a step forward in contributing new insights to this field by designing fraction proficiency tasks explicitly intended for formative assessment of students' comprehension of fractions. These fraction proficiency tasks were administered to a class of 35 fifth-grade students (ages 10–11) with mixed abilities in a primary school in Taiwan to evaluate their understanding of fractions. Findings of the study offer valuable insights into assessing students' understanding of fractions and provide a comprehensive view of the diversity in students' understanding and the extent of these differences. Implications for future studies are also presented.

Keywords: Fractions, fraction proficiency, assessment resources, assessment for learning, formative assessment.

Introduction

The teaching and learning of fractions persistently present challenges for both teachers and students. It has been argued that, for many students, learning fractions often involves merely manipulating symbols to arrive at the correct answer. While they might employ the appropriate fractional terms and solve some fraction-related problems, several critical aspects of fractions still escape them (Soni & Okamoto, 2020). Students' struggles with fractions often stem from the intricate relationships between various representations and fundamental arithmetic operations (Cramer et al., 2002), wherein the simultaneous symbolic nature of fractions contributes to these challenges. For mathematicians, fractions are rational numbers expressible in the form "a / b" where $b \neq 0$, rather than simply representing parts of wholes. They are not just ratios of two natural numbers but also constitute numbers in themselves.

In school, many children often receive only brief exposure to the concepts and procedures of fractions and are taught fraction algorithms with minimal emphasis on their conceptual underpinnings (Lenz et al., 2022). One of the conventional concrete approaches to learning about fractions often involves thinking in terms of partitioning or equal sharing. It is wise to base this idea on discrete countable objects, as well as on objects that may require the dissection of a continuous whole. However, this concrete approach does not cover the entirety of understanding fractions. For instance, the concept of "equal sharing" is just one among many properties of fractions and, on its own, is not adequate to convey a meaningful understanding of fractions to children.

Much research has concentrated on the challenges students encounter specifically within the field of fractions and their associated teaching and learning pedagogies, However, there has been insufficient and disproportionate attention given to assessment resources, especially for formative assessment, specifically designed for fractions. The significance of formative assessment, also referred to as assessment for learning, lies in its capacity to offer continuous feedback and insights into students' understanding and progress. This study has taken a step forward in contributing new insights to this field by designing fraction proficiency tasks explicitly intended for formative assessment of students' comprehension of fractions. In doing so, it seeks to provide valuable insights into the assessment of students' understanding of fractions, offering a comprehensive perspective on the diversity and extent of challenges encountered by students.

Fraction proficiency tasks

Drawing from Tsai and Li's (2017) fraction proficiency framework and an extensive review of fraction-related studies, 15 tasks were designed to assess students' comprehension levels and identify areas where they might encounter difficulties in the field of fractions. The content of these 15 tasks was specifically organized in four major topics to encompass the five dimensions of fraction proficiency identified by Tsai and Li (2017), which include: (1) the part-whole, measure, quotient, operator and ratio constructs of fractions, (2) the concept of equivalent fractions, (3) the procedural fluency for and conceptual understanding of fraction operations, (4) the relationship between fractions, decimals and percentages, and (5) the transition between different forms of representations involving fractions. Lesh's (1981) representation model (Dimension 5) was integral to all tasks that required students to solve problems by transitioning between representations. In the following sections, examples of tasks for each topic will be provided.

Topic 1: Five constructs of fractions

Topic 1, centered on five constructs of fractions (Dimension 1), involved designing four tasks aimed at assessing students' understanding of part-whole, measure, quotient, operator, and ratio constructs related to fractions. The statements for these four tasks are provided in Table 1.

Topic 2: Equivalent fractions

In Topic 2, focusing on equivalent fractions (Dimension 2), three tasks were formulated to explore students' comprehension of equivalent fractions and their conceptualization of expanding and reducing fractions to determine an equivalent form. Table 2 outlines these three tasks.

Topic 3: Multiplication of fractions

Topic 3, emphasizing the multiplication of fractions (Dimension 3), comprised five tasks designed to assess students' understanding of the reasoning behind their procedural skills in performing fraction multiplication. The statements for these four tasks are provided in Table 3.

Topic 4: Fractions, decimals and percentages

In Topic 4, three tasks were devised to examine the extent to which students recognize the relationship between fractions, decimals, and percentages (Dimension 4).

Table 1: Descriptions for tasks related to Topic 1

	Statement of Task	Transitions between representations (Dimension 5)
Topic 1 Five constructs of fractions (Dimension 1)	What is a fraction? How would you would explain to someone what a fraction is? Please offer three different explanations, and one or more of your explanations needs to relate to a real-life situation.	From the fractional symbolic representation to the real-life situation representation.
	Are they reasonable to you? (Students are given a set of cards that visually represent a fraction) Please select the cards you consider reasonable and explain your reasoning behind your choices.	From the pictorial representation to the fractional symbolic representation.
	Who spends more? Mary and John went to McDonalds. Mary spends 1/4 of her pocket money and John spends 1/2 of his. Do you agree it is possible that Mary spent more than John? Why do you think this?	From the real-life situation representation to the fractional symbolic representation.
	Which ones are reasonable? (Students are given a set of cards that visually represent a fraction) Please look at the cards provided; which of these cards are reasonable and which are not?	From the pictorial representation to the fractional symbolic representation

Table 2: Descriptions for tasks related to Topic 2

	Statement of Task	Transitions between representations (Dimension 5)
Topic 2 Equivalent fractions (Dimension 2)	What are equivalent fractions? (students are given a set of cards that visually represent an equivalent fraction). Please write down your observations from these cards, and then elaborate on how your findings are connected to equivalent fractions.	From the pictorial representation to the fractional symbolic representation
	Match the pairs (students are given a set of cards that visually represent an equivalent fraction) Please match equivalent fractions from these cards and then explain how you paired them.	From the pictorial representation to the fractional symbolic representation
	Who gets more? At two different tables where 2 children were sharing 3 chocolate bars and 6 children were sharing 9 chocolate bars. These chocolate bars are all the same size. Please indicate who will receive more and elaborate on your thought process behind your choice.	From the real-life situation to the spoken representation

Table 3: Descriptions for tasks related to Topic 3

	Statement of Task	Transitions between representations (Dimension 5)
Topic 3 Multiplication of fractions (Dimension 3)	Let's fold a paper fraction (Students are given pieces of colour paper) Please fold the fractions: 1/8, 1/6 and 1/12 using the paper provided.	From the fraction symbolic representation to the manipulative representation
	Jenny's birthday party Jenny wants to invite her three best friends to come to her party. Each of her three friends can consume 3/4 of a pizza Please illustrate with a diagram to show the quantity of pizza Jenny will require, and then provide a mathematical written representation to represent your drawing.	From the real-world situation to the pictorial representation
	What do you think 2/3 × 5? Please provide a real-life scenario that represents the mathematical operation 2/3 × 5 and then use a drawing to represent 2/3 × 5.	From the fractional symbolic representation to the real-life situation and to the pictorial representation
	How much cake had Jenny's brother eaten? Jenny's mum made a square-shaped cake for her birthday. At the party, half of the cake was eaten and then the rest was put in fridge. The next day, Jenny's brother ate 2/3 of the remaining part of the cake. (Students are given pieces of color paper) How would you fold the paper to illustrate the portion of cake Jenny's brother had consumed? Afterwards, provide a written mathematical representation to explain the folding method.	From the real-life representation to the manipulative representation
	What do you think 1/4× 3/4? Please provide a real-life scenario that represents the mathematical operation 1/4× 3/4 and then use a drawing to represent 1/4× 3/4	From the fractional symbolic representation to the real-world representation and to the pictorial presentation

Data collection and analysis

This study recruited a class of 35 fifth-grade students (ages 10–11) with mixed abilities from a primary school in Taiwan to evaluate their understanding of fractions. The four topics covered in the fraction proficiency tasks had been introduced to the participants in their previous school years as part of the current mathematics curriculum in Taiwan. The fraction proficiency tasks were given to all students without time limits for completion. Most students finished within 60 to 70 minutes and submitted their answer sheets and materials (such as cards and colored folding paper) to their teacher upon task completion.

Table 4: Descriptions for tasks related to Topic 4

	Statement of Task	Transitions between representations (Dimension 5)
Topic 4 Fractions, decimals and percentages (Dimension 4)	What is a percentage? Please explain what a percentage is and provide some examples from your life where percentages are commonly observed.	From the fractional symbolic representation to the real-life situation representation
	To what extent is Tom sure? When Tom is going to school, his mum asks him if he is prepared well for his school test today. Tom replies: "Yes". Mum asks: "Are you sure?", Tom says: "One hundred per cent sure". Mum asks: "So you will get a full mark home, will you?" Tom makes a funny face and says: "Um, um, fifty per cent sure". Please explain the meanings of "one hundred per cent sure" and "fifty per cent sure," and then demonstrate how fractions can represent these expressions.	From the real-life situation representation to the fractional symbolic representation
	How are they related? Here are three numbers: 0.4, 2/5 and 40%. Please explain the relationship between these three numbers and how they can be converted from one form to another.	From the symbolic representation to the spoken representation

Apart from the data collected from the students' drawings and paper folding exercises, which were neither numerical nor narrative, much of the data collected was in the form of words. An inductive coding approach was employed to identify both general and distinctive features from the texts, following these three steps: identifying and labeling, reducing, and summarizing. This aligns with Thomas's (2006) assertion that inductive approaches are designed to facilitate an understanding of meaning in complex data by developing summary themes or categories derived from the raw data.

The nature of students' fraction understanding

What can be learned from Topic 1?

This topic, based on Kieren's (1988) theory, assessed students' comprehension of the five constructs of fractions, revealing that their understanding of fractions was either confused or incomplete. Their grasp of fractions predominantly revolved around the part-whole construct, with minimal consideration for the equality of each part of the whole. This aligns with existing literature suggesting an excessive focus on the part-whole construct, hindering students' ability to position fractions on a number line (Saxe et al., 2013). Moreover, the challenge students faced in positioning 3/5 on a number line in this study further confirms the findings of Soni and Okamoto (2020), emphasizing students' struggles in locating fractions accurately on a number line. Another challenge observed was the students' inflexible recognition of a fraction's unit. Chan et al. (2007, p. 26) also argued that "the units concept is a common conceptual deficiency among students, indicating a significant flaw in current fraction teaching practices in Taiwan".

What can be learned from Topic 2?

An analysis of students' responses to the tasks in Topic 2 supported earlier research findings (Lamon, 2007) that students' reasoning of equivalent fractions was rather rule-based. For example, the "Who gets more" task in Table 2 showed that 20 out of 35 students answered it correctly. However, their explanations generally referred to "Because 3/2=9/6"; "Because they are equivalent fractions" or "By using the rule of expansion or reduction, you then know they are the same" to explain how they solved the task. It is not wrong to describe the equivalence of fractions based on the rules of expansion or reduction, but there is a danger that students apply "rule-based" explanations without understanding them (Levenson et al., 2004). This rule-based emphasis is also echoed in Yang's (2005) finding that both teachers and students tended to "rely on rule-based methods to explain their reasoning" in the field of fractions. This suggests the importance of allowing students the opportunity to articulate their thoughts and construct explanations that are not solely rule-based.

What can be learned from Topic 3?

An understanding of fraction multiplication often challenges students because they have to distinguish it from multiplication of whole numbers, that is, from repeated addition to multiplicative reasoning. An operator construct of fractions is fundamental for interpreting the meaning behind the multiplication of fractions (Thompson & Saldanha, 2003). In this topic, the "What do you think 2/3 × 5?" task (see Table 3) shows that multiplication of a whole number and a proper fraction was presented by more than half of the students as repeated addition (e.g., 2/3 × 5= 2/3 + 2/3 + 2/3 + 2/3 + 2/3). Such additive reasoning, although it provides a useful connection between multiplication and addition, may not be meaningfully interpreted for multiplying two proper fractions, which would produce a smaller fraction. This may explain why over half of students encountered challenges in providing a real-life scenario to represent the mathematical operation 1/4× 3/4 and to depict 1/4 × 3/4 through a drawing when responding to the "What do you think 1/4× 3/4?" task in this topic.

What can be learned from Topic 4?

In this topic, students' responses to the "What is a percentage?" task (see Table 4) highlighted their struggles in articulating their reasoning behind percentages. However, their responses to the "How are they related?" task revealed that 28 out of 35 students were capable of converting procedurally between these three different forms. This suggests that the students in this study recognised the quotient construct of a fraction – i.e. 2/5 means 2÷ 5 and 0.4 means 4 ÷10 – and, when dividing the numerator by the denominator, they had no problem converting from a fraction to a decimal. This proficiency contrasts with Moss's (2005) findings, where over half of the students (sixth and eighth graders in Canada) claimed that "1/8 would be 0.8" when expressed as a decimal. This emphasizes the critical role of the quotient construct in comprehending the connection between fractions and decimals. Moreover, the outcomes of this topic demonstrated students' proficiency in employing various strategies to convert between fractions, decimals, and percentages, suggesting they understood the relationships between three different forms that have identical values.

Fraction proficiency tasks for formative assessment of students' comprehension of fractions

Formative assessment takes various forms, aiding both students and teachers in evaluating learning objectives and adjusting instruction. Fraction proficiency tasks, as demonstrated earlier, can serve as formative assessments, enabling students to demonstrate their skills and identify errors and misconceptions in fractions. Within the classroom, these tasks can seamlessly integrate into ongoing formative assessment practices, allowing teachers to gain insights into student progress, deepen understanding of fractions, and address individual learning needs efficiently. Teachers observe student engagement, provide immediate feedback, and encourage self-assessment, fostering metacognitive skills and ownership of learning. Peer assessment can further enhance learning by providing diverse perspectives and collaborative feedback (Black et al., 2003).

Limitations, implications and directions for future research

This study focused on a specific mathematical area – fractions – and it only examined students' understanding of fractions based on the Tsai and Li's (2017) framework. It is recognized that various other aspects pertaining to fractions might not have been incorporated in these tasks; also, other related factors might not have been taken into account. The findings are confined by the constraints of the employed methodology as well as the limitations inherent in the sample. However, the fraction proficiency tasks presented in this study offer valuable insights into assessing primary students' understanding of fractions. They provide a comprehensive view of the diversity in students' understanding and the extent of these differences.

This study shows that the difficulties encountered by students in the present study resonate with those identified in previous studies. This suggests that fractions continue to pose challenges for students, even among Taiwanese students who are consistently recognized as high-performing in large-scale mathematics comparative assessments such as TIMSS and PISA. The findings of this study also offer assessment resources for teachers to gain a clearer understanding of what students should attain and what areas they need to develop. This assists in integrating diverse aspects of fraction knowledge, aiding both students and teachers in comprehending fractions more effectively.

Another implication of this study for fraction assessment involves reconsidering the role of assessment in contributing to a broader understanding of students' grasp of fractions and their mathematical knowledge overall. As argued by Saxe et al. (2013), fractions-related topics are often seen as disconnected. Therefore, further research is needed, particularly in formative assessment, where evaluating a comprehensive understanding of fractions across multiple facets should take precedence over isolating one facet from the others.

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