Design of tasks for assessing Diophantine equations on the new Mexican School

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Keywords: Diophantine equations, modeling, mathematical curriculum.

Linear Diophantine equations

In mathematics an equation is an algebraic statement in which it is shown that two amounts are equal using mathematical symbols like numbers, variable or unknown values joined by arithmetic symbols, and it is true for some values. This is an essential tool for modeling real life situations. Moreover, Amorim (2020) reminds us that one of the challenges in the initial training of mathematics teachers is to articulate the contents studied in the disciplines at the university with the themes of primary and secondary school. In this poster we focus on the Diophantine equations as they are part of the new curriculum 2022, National Strategy for the Improvement of Normal Schools, in Mexico.

Given polynomial equations with integer coefficients and which solutions are also integer numbers are called Diophantine equations. For now, we focus on linear equations, more precisely with two variables, i.e., equations like the following: $ax + by = c$, with a, b and c integers numbers.

Curriculum 2022 National Strategy for the Improvement of Normal Schools

Recently, the basic education curriculum in Mexico has been updated with the aim of covering the needs of the development of individuals. For this, the new curriculum 2022 in its approach of the mathematics teaching and learning degree states that:

> Mathematics is understood as a complex social and cultural construction; On the one hand, it is a set of heterogeneous knowledge in permanent construction, dynamic and situated, and on the other, it is a scientific discipline with its own knowledge production procedures, which must be learned and taught among new generations. Its value lies in the fact that it allows the subject to situate himself, order and understand the world. (Secretaría de Educación Pública [SEP], 2022, p. 1)

Below I explain the treatment given to the design of the tasks for the evaluation of the topic "Diophantine equations".

Tasks-design for future teachers on the topic Diophantine equations.

As we know linear equations with one variable are perhaps the simplest tool for modeling and solving problems, but what happens when more than one unknown value appears? Moreover, what happens when we try to design activities to contextualize them, where there is also implicit information that the individual destined to pose and solve a problem must identify to use it in the correct modeling of the situation? A problem proposed for a group of 2nd semester of teachers in training to try to answer both questions was: On the "Pozolito" farm there are ducks and dogs. Mr. Genaro told his grandson that counting only the legs of the animals there are, there is a total of 54. Write an equation that describes the situation and finds at least one solution (if it is possible). Explain your answer.

The students took 15 minutes for solving the problem. Answering the first question, the most common error that occurred was that the implicit information (number of legs of both animals) was identified but the same variable was used, so the equation $2x + 4x = 54$ was proposed, and they quickly responded that reducing similar terms, i.e., $6x = 54$ the solution was $x = 9$. However, they do not differentiate between how many of them are ducks and how many of them are dogs. Later we will see, that 9 has no sense with the correct answers. Now for the second question, common errors were also identified. In this case, several of the students identified that the number of ducks was a variable x, and the number of dogs another variable y, but they did not use the implicit information (number of legs of both animals), so they proposed the equation $x + y = 54$, giving a long list of possible solutions, which was obviously incorrect because the implicit information was really fundamental for the statement and subsequent resolution. These difficulties were also reported by Edo (2013).

Then, what was the answer to the problem? As a first step, formulate the correct equation, that is, $2x + 4y = 54$, and then proceed to solve it. Clearly, the first thing to do is check that the equation does indeed have integer solutions, which are the ones that interest us and that they are clearly solutions to the contextualized problem, because even when we know that the equation has an infinite number of solutions in real numbers, we are not interested in saying that there are 3.5 dogs. It is easy to verify that this problem has integer solutions.

Now, although in general we look for integer solutions for a Diophantine equation, and we have established that they are an infinite quantity, if they exist, for problems that model a situation of real life we must take in account which answers really are important to us, because in this case we are not interested in solutions like having -3 ducks either. Thus, the solutions reduce to $x = 1 + 2t, y = 13 - t, t \in Z, 1 \leq t \leq 12$.

The assessment of this problem, it was found that only a sixth of the group completely solved the task, and in total a quarter partially solved it, even intuitively, by trial and error. Due to all the factors observed, it is planned to replan the work with the contextualization of the contents, particularly with Diophantine equations, to improve future evaluations and the design of exercises paying attention in these and other factors that we can detect because they are an example of modeling simple real life.

Acknowledgment

To the Teaching Updating Center in Zacatecas for the supporting to this work.

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Secretaría de Educación Pública [SEP] (2022, 03 march). *Anexo 12. Plan de estudio de la licenciatura en enseñanza y aprendizaje de las matemáticas.* 9FUnbRJp85-ANEXO_12_DEL_ACUERDO_16_08_22.pdf (sep.gob.mx)