Conception of two informative tutorial feedback strategies for mathematical tasks with STACK

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The increasing presence of computer-based teaching and learning opportunities in recent years offers new approaches for providing informative tutorial feedback (ITF). For mathematical tasks, the tool STACK can be a possible approach. STACK enables the automated evaluation of even open mathematical tasks with any number of correct solutions and the provision of immediate feedback for learners. This paper will therefore investigate the question of how STACK could be used to design ITF-strategies for mathematical tasks. As a result of this question, two different strategies are presented, which differ when the cause of an erroneous answer is not clear. While in one strategy learners are provided with solution-specific hints for processing the task, in the other strategy they enter a task loop in which they must work out these hints on their own. Finally, the strategies are compared in terms of their characteristics and possible effects on students' learning processes.

Keywords: Computer assisted learning, education, informative tutorial feedback, mathematics

Introduction

Feedback plays a crucial role in mathematics education as it is one of the most powerful influences on improved academic achievement, with a high effect on student learning (Hattie, 2009). When designing feedback, it must be considered whether it is included in a formative or summative assessment. Summative assessments are provided at the end of the learning process and serve to evaluate the learning outcomes. Formative assessments, on the other hand, are provided during the learning process and are intended to have a positive impact on it (Pals et al., 2023). In this theoretical paper, formative assessments are focused on as the goal is to design feedback that supports learning processes.

In recent years, computer-based teaching and learning tools have offered new ways of providing informative tutorial feedback (ITF) (e. g. Erickson et al., 2020). This refers to feedback for digital learning environments that provide learners with solution- and error-specific hints rather than concrete solutions, enabling them to correct their incorrect answers independently. Especially in the field of mathematics education, the STACK tool represents a promising approach to provide learners with. STACK enables the automatic assessment of even open mathematical tasks and can provide learners with immediate feedback ITF (Knaut et al., 2022). This paper aims to investigate the use of STACK for the realization of effective ITF-strategies for mathematical tasks.

The theoretical background highlights the central role of STACK in automatizing the assessment process for mathematical tasks. With its integrated computer algebra system (CAS), STACK enables the evaluation of students' answers to various mathematical problems. In addition, this section describes the relevant characteristics of ITF-strategies. Subsequently, the research question of how STACK could be used to design ITF-strategies for mathematical tasks is addressed. To answer the research question, two different implementations of ITF-strategies (summarizing and guiding feedback) are presented and explained. The effectiveness of both strategies is discussed on a theoretical basis, highlighting their potential impact on students' learning processes. The paper concludes with a summary and an outlook on further related issues.

Theoretical background

STACK

Computer-based tools can be utilized to automatically evaluate tasks with even free input fields and to provide learners with feedback concerning their achievement. For mathematical tasks, this can be accomplished with STACK (System for Teaching and Assessment using a Computer algebra Kernel). Due to the implemented CAS, the digital evaluation of tasks with any number of correct solutions is possible (Alarfaj & Sangwin, 2022). Furthermore, emerging numerical values can be randomized. As a result, learners are offered numerous opportunities to practise by creating a single task. Meanwhile, STACK is available as a free plugin for Moodle, allowing the creation and use of STACK tasks in this learning platform.

For the digital evaluation and provision of feedback, a so-called response tree must be constructed in advance for each task (see Figure 1). Within the response tree, the learners' answers are checked for various mathematical characteristics in several nodes (Knaut et al., 2022). Through the underlying CAS, it is possible, for example, to check whether an entered function has certain zeros or extreme points. After a positive or negative result of the check, the path can be stopped or connected to another node for further analysis. In Figure 1, the paths are visualized by green and red lines. On each path, feedback can be stored and points for performance can be added or subtracted. Overall, the construction of a response tree enables the evaluation of various answers and the provision of feedback for learners (Sangwin, 2023).

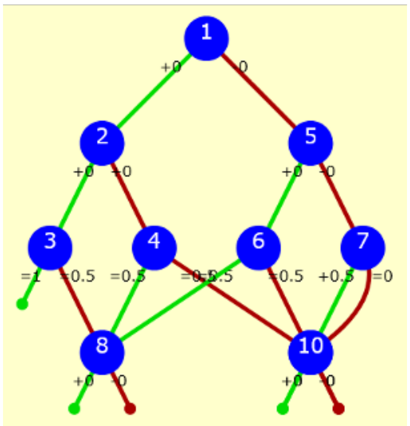

Figure 1: Example of a response tree for a task

Informative tutorial feedback strategies

Narciss (2008) has classified various forms of feedback in terms of their content. A distinction is made between simple and elaborated forms of feedback. While simple forms of feedback only include information about the correctness of a task, elaborated forms include more extensive information. Examples of simple feedback are *knowledge of performance* (KP), which indicates the number or proportion of tasks solved correctly, and *knowledge of result* (KR), which contains a specification of which tasks were solved correctly. An example of an elaborated form is feedback discussing certain errors and explaining possible causes, which is assigned to the category *knowledge about errors* (KM). If solution-specific hints are included in the feedback, it is referred to as *knowledge on how to proceed* (KH). Solution-specific hints contain information about strategies and necessary intermediate steps to solve the task without presenting a complete solution.

For digital learning tools such as the STACK system presented previously, a feedback strategy can be used to define which feedback forms are combined and presented to learners. Accordingly, a feedback strategy is a defined plan that determines the structure and presentation of feedback (Narciss, 2006). One approach is to develop ITF-strategies, which combine simple KR-feedback with elaborated forms like KH- and KM-feedback (Narciss, 2012). Instead of giving learners correct answers, the focus is on enabling them to use the feedback to correct their responses. As a result, learners should be allowed to retry tasks immediately after receiving feedback. The opportunity for immediate implementation increases the relevance of the feedback for the learners and leads to a more intensive engagement with it (Tärning et al., 2020). This aims to support the students' learning processes.

Research question

Regarding the support of learning processes through ITF-strategies, the question arises of how these strategies could be implemented for practical use with current technologies. There are already a few studies on the implementation of ITF-strategies in digital learning environments for specific mathematical areas such as written subtraction (Narciss & Huth, 2004). However, research is needed on how ITF-strategies could be implemented in general for mathematical tasks. This research is essential for a wider use of these strategies in mathematics classes and for the investigation of them in experimental studies. The STACK system could provide a possible approach to this need. This paper will therefore investigate the research question of how STACK could be used to design ITF-strategies for mathematical tasks. To answer the research question, two possible approaches using STACK are developed, which both respect the presented characteristics of ITF-strategies. The two strategies are described in detail in the next chapter.

Conception of ITF-strategies with STACK

First, we consider the two cases in which either a correct solution or an erroneous solution is entered where the cause of the error is recognizable. The design of the two strategies is identical for these two cases. In both strategies, students receive KR-feedback in the event of a correct input. The correct solution can be verified in STACK in two different ways due to the underlying CAS. Firstly, the answer can be checked for algebraic equivalence using a stored sample solution. On the other hand, the entered solution can also be tested for relevant characteristics. These options ensure that every correct answer is recognized, even if it is entered in a different form (e.g. fraction instead of a decimal number) or if there are any number of correct answers.

Both strategies offer KM-feedback for certain incorrect answers. This requires appropriate checks to be carried out in advance in the response trees. Due to the CAS, it is sufficient to test the submitted answer for relevant mathematical characteristics and hence no explicit incorrect solutions need to be stored in the response trees. For example, it is possible to check whether an inserted function has the required degree or whether the correct variable has been used. In this way, many incorrect answers, which are all due to the same causes of error, can be recognized in some individual nodes. In this way, differentiated KM-feedback can be formulated for several typical errors.

The described common components of both strategies are illustrated in Figure 2 in the form of a diagram for an exemplary integral task. As stated previously, KR-feedback is displayed if the input is correct. In the event of an incorrect answer, several different error-specific hints have been stored where the cause of the error can be deduced reliably enough to provide KM-feedback. For the sake of clarity, not all five KM-feedback cases are listed in full in Figure 2, but only two of them.

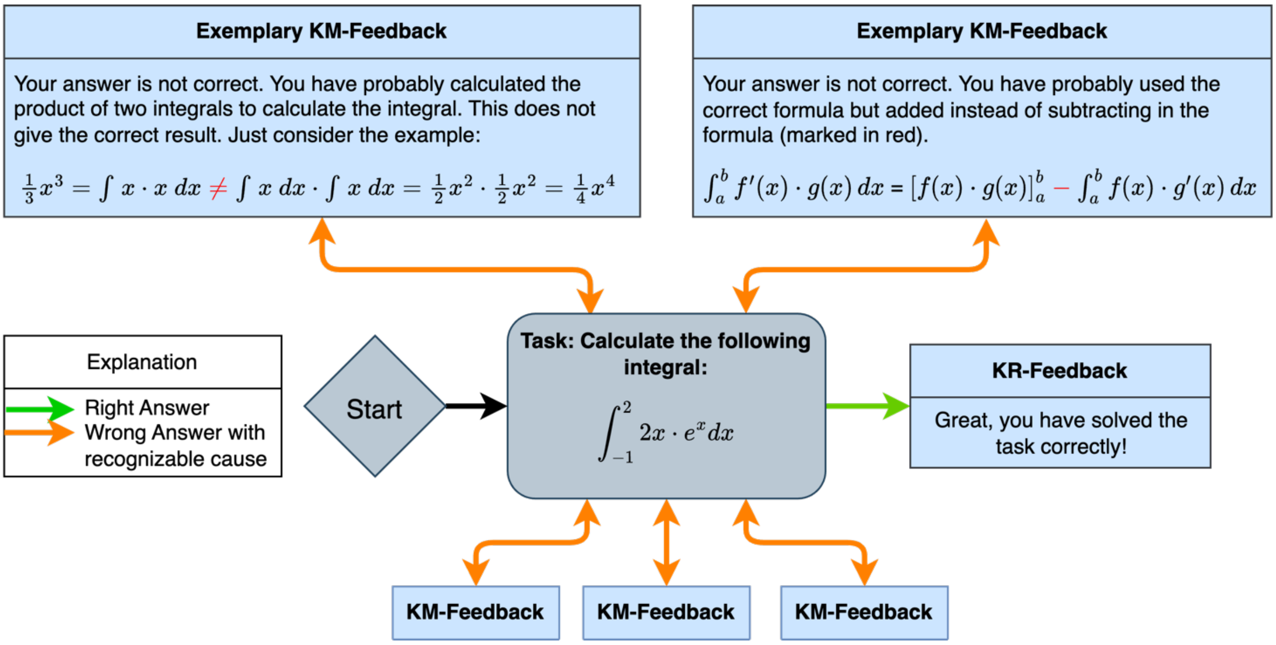

Figure 2: Common feedback components of both strategies in an integral task

In the first case shown (see top left in Figure 2), feedback is given if a solution has been entered that arises when the product of two integrals has been calculated instead of the integral of the product. This scenario utilizes an example to explain why this method is typically not allowed. The second case (see top right in Figure 2) deals with the situation where the correct approach of partial integration has been chosen but the terms in the formula have been added instead of subtracted. Here the formula is shown, and the minus sign is highlighted in red.

Now the challenging question arises of how to deal with certain incorrect entries for which no specific error causes have been created in the response tree. This is a likely scenario, as task creators cannot consider every possible cause of error in advance. In addition, particularly in the case of more complex tasks, a combination of several error causes can result in incorrect answers that cannot be recognized based on the final solution. Nevertheless, it must be ensured that students receive helpful feedback even in the case of such entries to recognize their errors and correct their solutions. The two feedback strategies deal with this situation differently.

As part of the first feedback strategy, students receive KH-feedback in the event of an incorrect answer that was not saved in the response tree, which contains solution-specific instructions for processing the task. However, the correct solution is not anticipated, as the students are asked to use the information presented to perform the last step independently. In this way, learners still have the opportunity to correct their answers. We describe this strategy as *summarizing* feedback.

This case is also illustrated in Figure 3 for the integral task. We assume that an incorrect answer was given that cannot be assigned to any of the five possible causes of error from the previous figure.

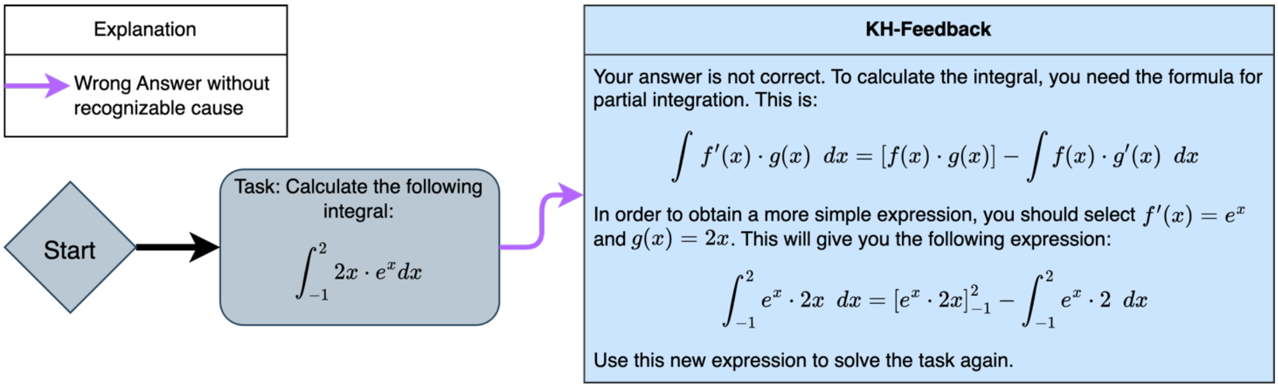

Figure 3: KH-component of the summarizing feedback in an integral task

In this case, the hint and the formula for partial integration are given in the KH-feedback (see Figure 3). Furthermore, the choice of functions for the two factors and the resulting expression are presented. It is now the learner's task to calculate this new integral expression.

However, the second feedback strategy handles this case differently. In the event of an incorrect answer, for which the cause of the error is not stored in the response tree, the second feedback strategy does not provide any error-specific feedback. Instead, students can enter a task loop in which they work their way through the entire task in a series of sub-steps. In these task loops, the solution-specific hints from the summarizing feedback are now reformulated as questions which now should be answered by the students. They also receive error-specific feedback while working through the sub-steps. This feedback is usually more extensive than the feedback received when working through the initial complex task. The reason for this lies in the fact that more precise insights into the causes of errors can be drawn from the answers to the sub-steps. We refer to this strategy as *guiding* feedback.

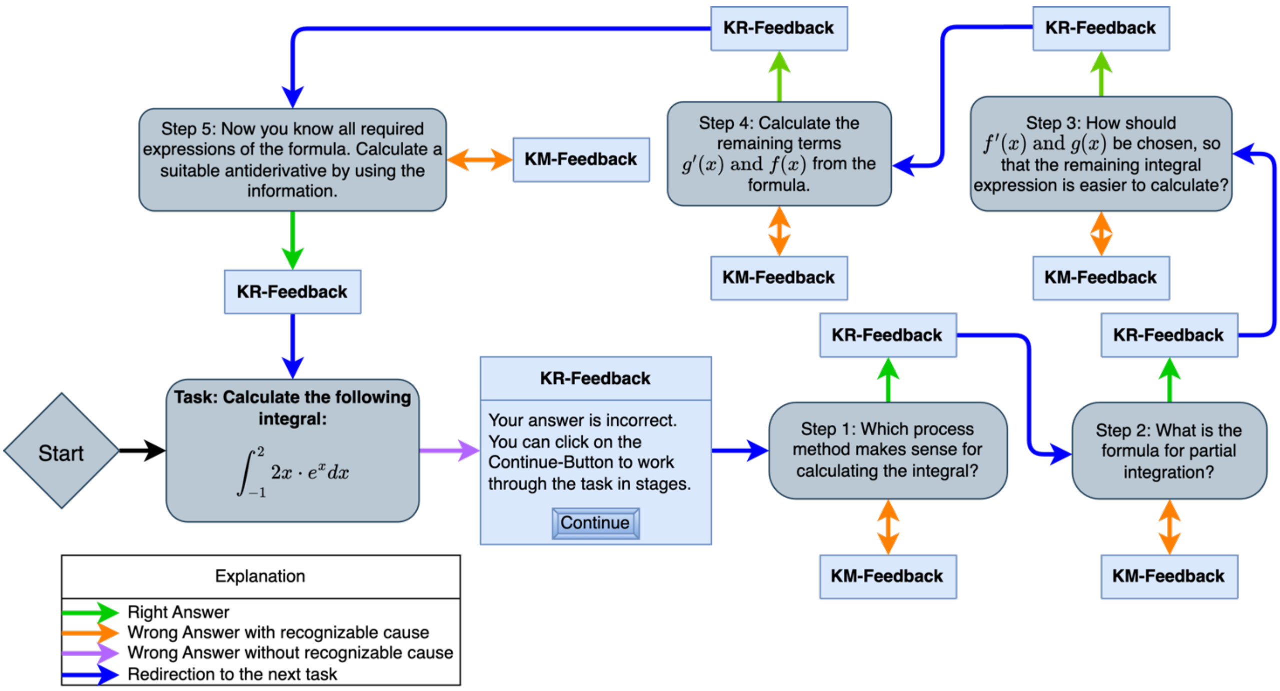

Figure 4: Task loop of the guiding feedback in an integral task

In Figure 4, the case in which an incorrect response was entered that cannot be traced back to the cause of its error is now visualized for the guiding feedback. Concerning this situation, a button is displayed in the feedback that leads to the task loop (see Figure 4). The first sub-step relates to the necessary method for calculating the integral. In a drop-down menu, the correct solution (partial integration) among some distractors can be selected. If the answer is incorrect, KM-feedback is given for each answer option, explaining why the selected option is not appropriate or not permitted. If the answer to the first sub-step is correct, KR-feedback appears, which leads to the next sub-step. While the second sub-step still has a closed answer format, the third, fourth, and fifth sub-steps have open input fields. For these three open input fields, a total of ten error-specific hints are provided if required. They are not explicitly shown in Figure 4 for reasons of clarity. In the fifth sub-step, learners are asked to specify a suitable antiderivative. Once this last sub-step has also been successfully completed, students return to the initial task. Now they can work on this task again using the sub-solutions they have worked out. Their sub-solutions are displayed so that they can easily access them. The task loop described can be repeated if needed.

The creation of these task loops is not a regularly offered feature of STACK. This option was developed at the Ruhr University Bochum by incorporating an additional JavaScript (Altieri et al., 2020).

For both strategies, arguments can be found that explain their potential contribution in supporting students' learning processes. Both strategies contain at least one important elaborated component in addition to the evaluative one (Kulhavy & Stock, 1989). Moreover, both strategies encourage learners to become active and correct their answers independently (Nicol, 2019). In both cases, this could lead to a more intensive engagement with the feedback and thus have a positive effect on the learning process (Tärning et al., 2020).

One favor of summarizing feedback lies in the fact that solution-specific cues target correct solution strategies and could therefore have a more motivating effect (Fong et al., 2018). In addition, similar to the use of correct solutions (Renkl, 2014), the presentation of solution-specific hints can be assumed valuable. Solution-specific hints draw students' attention to the correct solution procedure, making it easier for them to focus on acquiring new knowledge (Große & Renkl, 2007). Furthermore, its design could contribute to a clearer overview when working on the task and therefore lead to a better understanding.

Nevertheless, it can be argued that the guiding feedback has a stronger effect on the learning process, as the students are required to understand and apply the content of the feedback within the task loops to an even greater extent. The individual sub-steps that are displayed in the summarizing feedback must be worked out independently in the tasks with guiding feedback. This could have for example a direct influence on the self-efficacy of the students, as in this way more mastery experiences are perceived (Bandura, 1998; Ramdass & Zimmerman 2008). Another advantage that results directly from the task loops is that errors can be traced back more precisely to their causes when working through the sub-steps. In this way, the feedback can address the learning status and help students closing learning gaps (Hattie & Timperley, 2007).

Conclusion and outlook

The presented tool STACK is appropriate for the creation of tasks with formative feedback due to its many technical possibilities. The response tree in STACK can be used to provide learners with elaborated feedback on their answers. For this purpose, task creators must consider in advance, based on literature or empirical observations, which mistakes learners might make. They can then create response trees with appropriate checks and corresponding KM-feedback. However, the causes of errors can only be anticipated in advance to a certain extent. To also handle incorrect answers that cannot be traced back to specific errors by defined response trees, two feedback strategies were presented in this paper. In the summarizing feedback, solution-specific hints are presented. Despite this, in the guiding feedback, only error-specific hints are given, while intermediate solutions must be worked out in sub-steps.

The question of whether one of these feedback strategies offers greater support for students' learning processes requires empirical investigation. On the basis of theoretical considerations, reasons could be identified for both feedback strategies explaining their potential support for learning processes. Comparative studies are necessary to explore this further. Certainly, the effectiveness of the feedback strategies also depends on other factors to be investigated, such as the topic or the difficulty of the task and the individual prerequisites of the learners. A final decision between the two strategies may not be required by task creators, as strengths of the two feedback strategies could differ in cognitive, motivational and metacognitive areas.

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