Peer assessment in an undergraduate geometry course: Fostering proof competency in teacher students

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Proofs are integral to mathematics as a science, but they are difficult to learn and provide challenges for instruction. This is a particular problem in teacher education, where proofs are a topic that many students experience being disconnected from teachers' day-to-day work in schools. In this note, we report on a course development and research project exploring the use of peer assessment as a tool to foster proof competency in pre-service teacher students.

Keywords: Assessment, teacher education, mathematics education, proof competency, mathematical proofs, peer assessment

Introduction

Proofs are both a defining product of mathematicians' activity and a notoriously difficult topic for undergraduate students of mathematics. The usual mathematics curriculum in schools puts little emphasis on proofs, arguments, and formal explanations which is a source of tension for the design of courses for pre-service teachers (PSTs). On the one hand, students need to acquire competency in producing and understanding proofs and arguments; on the other hand, the level of formality of tertiary education proofs is often viewed by PSTs as irrelevant to the practice of teaching in schools.

In this article, we present a course development project aimed at training PST's proof competency and changing their attitudes on the topic. Our method to attain these goals is to use peer assessment. The study detailed in this paper is set to commence in early 2024. Note that this paper therefore will not feature results or data from this ongoing study. We anticipate sharing preliminary findings at the FAME conference. The design for the study draws inspiration from an earlier study which we conducted with the aim to improve computational skills of PSTs through peer assessment. Our overarching aim is to develop peer assessment as a tool to enhance *proof competency* (refer to the theory section for the definition of this term used throughout the paper) of the PSTs. The present article more modestly restricts to the following research questions:

1. To what extent does peer assessment contribute to the improvement of proof understanding and proof construction skills among PSTs participating in an undergraduate geometry course?
2. How effectively can PSTs assess the clarity and logical soundness of proofs and arguments generated by their peers during a peer assessment activity?

To this end, we incorporate peer assessment into a geometry course for teacher students. In the implemented activity, the PSTs are asked to evaluate each other's mathematical argumentation. Our interest here lies in the effects of peer assessment on proof understanding and construction abilities of the PSTs. Note that the evaluation of mathematical arguments is typically an unfamiliar task for PSTs in Norway.

Theory and background

Assessment and feedback can be effective tools teachers can use to promote students' learning (Hattie, 2008). For this study we are interested in peer assessment as a tool. Topping (1998, p. 250) defines peer assessment as being an arrangement in which students evaluate the work of peers of similar status. In a previous study (Julien, Romijn, Schmeding 2023) we investigated how peer assessment, in particular giving and receiving of feedback, enhanced mathematical knowledge of PSTs. There, assessment activities which involved giving feedback were shown to have potential to both enhance students computational and professional skills. These two objectives can be related to Shulman's distinction between pedagogical content knowledge (PCK) and subject matter knowledge (SMK), see e.g., Berry et al. (2016). We focus on assessment in mathematical tasks which address SMK as in mathematical competency but not PCK. In the literature, the effect of peer evaluation on PCK for pre-service mathematics teachers is discussed in Ayalon & Wilkie (2021). In contrast, there is little research on the influence of peer assessment on SMK in mathematics teacher education.

The purpose of the present paper and the associated study is to investigate the effects of peer assessment on proof understanding and construction in an undergraduate geometry course. As Lin et al. (2012) stress, teachers professional learning of proofs and teaching proofs depends on their knowledge, practice, and beliefs about proofs. Our main goal is to investigate how peer assessment among PSTs can be used to develop those aspects.

So far in this article, the term "proof competency" was used as an umbrella term which has not yet been defined; we shall remedy it now. The teaching of mathematical arguments and proofs in higher education and in teacher education has been an active research subject for quite some time. With a view towards teacher education, there are three different aspects to be considered: knowledge of proof, practice of proof and beliefs about proof. These are interdependent and need to be addressed simultaneously to improve proof competency (Lin et al., 2012). For our study, the construct of proof competency consists of four related aspects (see Selden and Selden, 2015): proof comprehension, proof construction, proof validation and proof evaluation.

Proof comprehension is the ability to read and understand written proofs. The "big difference" (Selden and Selden 2015, p. 4) between proof comprehension and proof validation is that in a proof comprehension situation, it can be assumed that the presented proof is correct, while this is not the case in proof validation situations. The distinction is of particular importance for us, as PSTs train for situations in which they are asked to validate and evaluate arguments and proofs. Proof evaluation describes the assignment of a value judgement to a proof (attempt). For professional mathematicians this often means judging a proof on its merits of conveying ideas and concepts. We view it as equally important for PSTs to be able to assess the presentation and clarity of a proof.

The three aspects of proof competency described in the last paragraph have in common that they apply to proofs presented to the PSTs. In contrast, proof construction asks for the creation of new proofs, usually to a statement provided to the learners. This activity in general is inherently more challenging than the other aspects and often requires substantial SMK. However, in a sense also the PCK is called upon in the construction and presentation of arguments to convince the reader of the validity of a claim. This social dimension can be viewed through the lens of communities of practice (here the PSTs in the course), see e.g. Selden (2012, section 3.1.2). Proofs need to provide an acceptable level of conviction that the mathematical statement is true. Following Mason et al. (1982) these levels are (in ascending order of sophistication): 1. Convince oneself, 2. Convince a friend, 3. Convince a sceptic. What is viewed as sufficient to qualify for the different stages in the model will depend on the social norms and practices within the community for which the proof is constructed.

Methodology and setting

The setting in which we will carry out the peer assessment is a geometry course in a large Norwegian university, which has a particular focus on axiomatic constructions. One of the main learning goals of this course is to revisit classic geometric results from a higher standpoint. This includes explicit proofs in an axiomatic setting. Thus, the course aims at developing students' geometric understanding as well as their understanding of and ability to produce proofs. The main stated public for this course is pre-service high-school teachers in their second or third study year.

In one of the assignments which the students need to hand in, they will assess an educator-made proof (based on student deliveries from a previous iteration of the course). This preparation task was selected to display subtle aspects of proving, which students usually find difficult, such as the need to prove that a condition is both necessary and sufficient. Two actual "peer assessment events" will then be carried out during the semester. Such an event consists of the following: students will solve a task knowing that it will be assessed by another student, and hand it in. In the next homework assignment, they will receive one of their peers' solutions from the previous assignment and assess it. For this, the PSTs will be given a grading guide. Our design aim for the guide is to strike a balance between general and specific instructions: the guide needs to be specific to provide scaffolding for the assessment; nevertheless, it should also not be a step-by-step solution as we want students to exercise their own judgment and autonomy in assessing statements and justifications.

To answer our research questions, we will evaluate both the quality of the students' proofs and the product of the peer assessment of these proofs. For the second research question, we will focus on clarity and logical soundness of arguments, as reflected in the proof and in the peer assessment of the delivery. We will then compare our own assessments with those obtained by the students in the peer assessment process. In addition, we will conduct interviews with participants. This qualitative data together with the results of a formal written assessment of the proof understanding and construction (conducted three times during the semester) will allow us to create a holistic description of the PSTs abilities. The assessment of proof understanding and construction is also part of a second research project focusing on the influence of learning videos on proving skills. The results from the assessments and additional interviews will shed light on the first research question.

Discussion

In designing our peer assessment experiment and data collection, we needed to clarify our goals: a large aspect of learning proofs is a question of students' autonomy. As Robert and Schwarzenberger point out "tertiary students need to learn to distinguish between mathematical knowledge and meta-mathematical knowledge of the correctness, relevance and elegance of proof and take responsibility for their own mathematical learning." (cited from Guzman et al., 1998, p.755). With that in mind, we believe that students having the responsibility of establishing the correctness of a peer's proof might be of value. In addition, reading the grading guide will make explicit and visible to them the set of demands that mathematicians make on what can be called a "proof". This relates to the "enculturation" aspect of learning proofs: evaluating other people's reasoning is an authentic activity both for mathematicians and mathematics teachers.

We are convinced that peer assessment activities can be developed to become a valuable tool for the acquisition of proof competency and the professionalization of teacher students.

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