Leveraging formative assessment to develop students' mathematical reasoning through images and student generated language

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A sixth-grade teacher who taught at a highly diverse school in the United States participated in professional development activities during the 2022-23 school year to learn about and implement an instructional protocol designed for use during problem-solving lessons. The protocol is intended to guide teachers to simultaneously attend to developing their students' mathematical reasoning and learning of the mathematics register. In a case study undertaken to examine teachers' use of the protocol, an example emerged of how formative assessment can be leveraged to support the development of students' mathematical reasoning through student generated language and images. This study contributes to the research literature by demonstrating how the use of formative assessment can promote the use of language to "carry" a mathematical concept in the sense that language can help students create a mental image of that concept.

Keywords: Formative assessment, instructional innovation, mathematical reasoning.

Introduction

During the 2022-23 school year, the lead author collaborated with "Ms. Diaz," a sixth-grade teacher who taught at a diverse school district in northern New Mexico, USA. Ms. Diaz volunteered to learn about and use an instructional protocol designed specifically for use during problem-solving lessons. The protocol, referred to as the "Discursive Mathematics Protocol" (DMP), is intended to be used as a guide by teachers to support students to develop both their mathematical reasoning and learn the mathematics register. Additional information about the DMP can be found in Matute (2022). In this paper, a vignette is shared from one of the problem-solving lessons in which Ms. Diaz implemented the DMP. The vignette demonstrates how formative assessment can be leveraged to support the development of students' mathematical reasoning through student generated language and images. The following research question is addressed in this study: What is an example of how formative assessment can used to support students' mathematical reasoning through student generated language? Brief reviews of the research literature on formative assessment and the role of language and images in the learning of mathematical concepts are now provided.

Formative assessment in mathematics

Classroom assessments are used to inform teachers, students and parents about student knowledge and understanding of mathematical concepts, processes and skills (Wiggins, 1993). There are two categories of classroom assessments; summative and formative. Summative assessment formats focus on what students know at a given time (Guskey & Bailey, 2001). Formative assessments differ from summative assessment in that the focus is not just on summarizing students' learning, but on using student learning data to inform instruction. After examining 250 research studies on classroom assessments, Black and Wiliam (1998) found that when teachers focus on formative assessment, student achievement gains are among the largest ever reported for educational interventions. Formative assessment can include any of the following: classroom observation, inquiry, group work, whole class discussions, peer assessment, written work, individual interviews, student self-assessment, and portfolio assessment (Gearhart & Saxe, 2004). The vignette provided below resulted from the interplay of classroom observation, inquiry, group work, and a whole class discussion.

The role of language and images in the learning of mathematics

Words are referents to mental images (Arnheim, 1969) that teachers can leverage to support their students' mathematical reasoning and learning (Gonzales, 2004). As one example, Kieren (1988) described how students used visual images associated with cutting, symmetry, and numerical halving to make sense of and express their ideas about the notion of something being a half of a whole. The deliberate use of images and language is particularly relevant in problem-solving based lessons given that representations are used to understand problems and devise solutions to those problems (Pólya, 1945/1986). In addition, students engage in language-rich activities to problem-solve by making conjectures, conceiving arguments, and formulating and carrying out proofs (Schoenfeld, 2013). The study's theoretical perspective is introduced next followed by an explanation of the research methods.

Theoretical framework

Given the considerable attention given to cognition, sense making, and social discourse in the DMP, our theoretical framework is social-constructivism (Shepard, 2000). In this study, we viewed cognition through a measurement lens as students solve a task introduced below, examined students' mathematical reasoning as they made sense of images and used language to devise solution strategies and test those strategies (Shepard, 2000), and situated student learning as occurring through their engagement in discourse in communities of practice (Erath et al., 2021).

Methods and data sources

This study was part of a case study research project that examined the DMP with teachers such as Ms. Diaz. In case studies, a real-life, bounded system or entity is selected to study within a particular setting (Yin, 2014). Throughout the 2022-23 school year, the following data was collected: observation data, videos of teachers and students who provided consent, student work and interviews with the two participating teachers. The data presented here were collected during one problem-solving lesson led by Ms. Diaz with her sixth-graders. A co-teaching approach (Cook & Friend, 1995) was used to plan and deliver the task. During the implementation of the task "Height Requirements" (See Figure 1), the lead author facilitated whole class discourse with Ms. Diaz as shown below.

At Sea World San Diego, kids are only allowed into the Air Bounce if they are between 37 and 61 inches tall. They are only allowed on the Tide Pool Climb if they are 39 inches tall or under: 1. Represent the height requirements of each ride in words.	3. Show the allowable heights for the rides on separate number lines. 4. Using inequalities and a number line, describe the height of kids who can go on both the Air

2. Represent the height requirements of each ride with inequalities. Explain the meaning of the terms you used for each of your inequalities.	Bounce and the Tide Pool Climb. Explain how you figured this out.
Adapted from IM: https://tasks.illustrativemathematics.org/content-standards/6/EE/B/8/tasks/2010	

Figure 1: Height Requirements Task

We used social-constructivism (Shepard, 2000) to examine students' mathematical reasoning in conjunction with how language was generated by Ms. Diaz and her students to communicate ideas. As will be shown, a vignette emerged as students were attempting to address the task's fourth question which involved determining the range of kids' heights that would allow them to go on two rides; the Air Bounce and Tide Pool Climb.

Results

In a video vignette described below, the phrase "in between" became a focus of instruction. What inspired the use of this phrase was that "Elena" used it when describing the shaded portion of her number line between 37 and 39 in a small group discussion with her peers and the first author. She shared, "It could just go in between 37 and 39." After a short discussion in which Elena explained further, another girl in her small group explained why the answer must be in between 37 and 39: "Because 37 is how far you have to be for the Air Bounce and 39 is how tall you have to be for the Tide Pool." In a whole class discussion, students in the class were asked to revoice Elena's idea. The following discussion ensued between two students and the lead author:

1	S1:	What she's basically saying is, like, you know how the Tide Pool ends at 39 and the Air Bounce starts at 37?
2	RK:	Yeah.
3	S1:	You say 37, 38, and 39 inches could ride both the rides.
4	RK:	Kids who are 37, 38 or 39 inches tall?
5	S1:	Yeah.
6	RK:	What if the kid is 37 and a half inches tall?
7	S2:	They can still ride it.
8	RK:	Why? Are they still in between 37 and 39 inches tall?
9	S1:	Yeah.

At this point, Ms. Diaz encouraged "Luna" to come forward to share her ideas at the front board. Luna described her double number line answer that shows an overlap of heights that would allow children to take both rides. Luna then proceeded to modify her written response to the task's fourth question displayed on the board by including the phrase "in between" in her response, observed by the entire class. Others leveraged this phrase as well in their written solutions to the task's fourth question, helping them derive the range of kids' heights that would allow them to go on both rides.

Discussion

The use of formative assessment, specifically an observation made of student generated ideas, provided the means to engage students in a whole class discussion on the language they produced to solve the task. In the given vignette, a student's use of the phrase "in between" led to students making sense of the meaning of the overlap of the two number lines. The overlap of two shaded sections of a number line allowed students a visual means to derive the solution to the task. The vignette shows how words are referents to mental images (Arnheim, 1969) and how teachers can leverage both images and language to help students learn mathematics (Gonzales, 2004). Moreover, the vignette reveals how language can "carry" a mathematical concept in the sense that language can help students create a mental image of that concept. Leveraging a simple phrase such as "in between" demonstrates how student generated language can support mathematical understanding. This study contributes to the research literature by giving an example of how the use of formative assessment can promote the use of language to "carry" a mathematical concept to help students create a mental image of concepts.

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