Fostering technology-enhanced formative assessment in Euclidean

geometry proving through graded peer tutoring roles

Annamaria Miranda¹ and Loredana Saliceto²

^{1,2} University of Salerno, Department of Mathematics, Italy;

¹amiranda@unisa.it and ² lsaliceto@unisa.it

In recent years, educators and researchers have paid increasing attention to formative assessment strategies. In this paper, we explore an experimental design in which formative assessment strategies intertwine with digital technology and graded peer tutoring. The activities developed within structured groups aim to overcome secondary school students' difficulties in proving Euclidean geometry statements. According to the design, each group is decomposed into three helping students, acting as guides at various levels, and one student needing to be guided to learn. Each helping student intervenes to activate a specific assessment process in a specific phase of the activity, supported or not by a digital tool, depending on the peer tutoring role to be performed. We investigate the use of digital technology and tutoring roles in supporting agents (teacher, students, peers) to develop formative assessment strategies in teaching and learning Euclidean geometry.

Keywords: Formative assessment, digital tools, roles, graded peer tutoring, proof.

Introduction and conceptual background.

Formative assessment (FA) is widely regarded as one of the more effective instructional strategies employed by teachers, with a growing body of literature and academic research on the topic (Sadler, 1998; Roschelle & Pea, 2002; Irving, 2006; Wiliam & Thompson, 2007; Black & Wiliam, 2009; Swan & Burkhardt, 2014). FA refers to a wide range of methods used by teachers to conduct inprocess evaluations of student understanding, learning needs, and learning progress during a lesson or course. What distinguishes an evaluation as *formative* is how it is used, i.e., as a method in which

evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (Black & Wiliam 2009, p. 7)

Research into FA practices has particularly highlighted the role played by so-called *connected classroom technologies* (CCT), networked systems of personal computers or handheld devices specifically designed to be used in classrooms for interactive teaching and learning (Irving, 2006). CCT supports FA due to their specific features that make them effective tools for FA in accomplishing the following: (1) *monitoring students' progress, collecting the content of students' interaction over longer timespans and over multiple sets of classroom participants;* (2) *providing students with immediate private feedback, keeping them oriented on the path to deep conceptual understanding* (Irving, 2006); (3) *encouraging students to reflect and monitor their own progress* (Roschelle & Pea, 2002). Cusi et al. (2017) designed and implemented CCT-supported digital resources, the *worksheets,* to activate FA processes during classroom mathematics activities. The overall goal of FA is to collect detailed information to improve instruction and student learning while it is taking place. We design an activity in which digital technology and metacognition support agents in activating FA processes to overcome students' difficulties in proving a statement in Euclidean geometry. A primary goal in secondary school is to have students become proficient at writing proofs in Euclidean geometry. We refer to formal proofs within the Euclidean axiomatic system as deductive arguments showing that

the assumptions of a statement logically guarantee the conclusion. However, this goal is rarely met. Many causes for students' difficulties in proving seem to depend both on how to start a proof and how to bridge the gap between informal and formal reasoning (Moore, 1994; Weber, 2001). Research has also shown that using methods of informal reasoning, including visual representations, can have a positive effect on the outcome of students' proof-writing processes (Raman, 2003). Visualization of diagrams sketching the statement of a theorem and the production of arguments, even if they are not mathematically rigorous, can lead to identifying the key idea of the proof (Raman, 2003; Zazkis et al., 1996), and the key idea begins to construct a bridge between argumentation and proof. Some researchers argue that experiencing 'cognitive unity' between conjecturing and proving can help students bridge the gap between empirical investigation/conjecturing and proving (Boero et al., 1996). The effectiveness of starting to prove a statement in Euclidean geometry from a visual representation is amplified if the diagram is drawn in dynamical geometry environments (DGEs). The literature regarding the use of DGEs in proof-related activity has paid attention to conjecture generation and the transition from conjecture generation to proof production (Baccaglini-Frank & Mariotti, 2010). Many studies have been conducted to investigate the affordances of DGEs, which include dragging and measuring modalities that result in the generation and testing of hypotheses by generating various diagrams (Arzarello et al., 2002; Olivero & Robutti, 2007). A statement to be proved in secondary school Euclidean geometry is frequently described with reference to a specific drawn diagram representing a certain general class. A diagram, on the other hand, may represent only one case and thus not capture all the configurations to which the statement may refer. As a result, a diagram-based deductive proof may be valid only in that case, and different proofs may be required for different configurations. DGEs can play a significant role in this type of generalisation because their dragging function allows for easy access to multiple diagrams while maintaining the geometrical relationships imposed on the diagrams. Our approach to achieving an efficient FA when facing a Euclidean proof intends to exploit the benefits DGEs can give students at the beginning phase of a proving process and to do this by actively engaging them in a peer tutoring setting. Moreneo & Duran (2002) describe peer tutoring (PT) as a method of cooperative learning based on the creation of pairs of students with an unbalanced relationship; that is, the tutor and the student needing help do not have equal competencies, but they share a common goal. The method can be the most intellectually rewarding experience of a student's career and serves as an effective way to improve self-esteem (Annis, 2013). From a Vygotskyan perspective (Vygotsky 1978), our design considers that socialisation experiences that occur during peer tutoring can benefit both the tutor and the needing student by encouraging students to learn at various levels: the interaction with more expert peers plays a crucial role in students' learning while the tutoring role develops metacognitive competencies. This involves the development of planning, monitoring, and critiquing behaviours, all metacognitive aspects (Schoenfeld, 1992) on which FA must be focused.

In this paper, we report the design and implementation of a cooperative learning activity that engages secondary school students in interacting according to a peer tutoring relationship framework we define in the next section. The model, based on the graduation of peer tutoring by roles, aims to foster FA processes and help students overcome difficulties in proving Euclidean geometry statements.

Theoretical framework and research questions.

The theoretical framework for our design, implementation, and analysis of the activity finds its roots in the combination of the use of technology to enhance FA practices, through the three-dimensional model of FA (Cusi et al., 2017), and the role of peer tutoring in activating FA processes. Wiliam & Thompson (2007) introduce five key strategies for FA practices in school settings (WT strategies): (a) *clarifying and sharing learning intentions and criteria for success;* (b) *engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;* (c)

providing feedback that moves learners forward; (d) activating students as instructional resources for one another; (e) activating students as the owners of their own learning. The three-dimensional model (Cusi et. al, 2017) considers: the five FA key-strategies described by Wiliam & Thompson; the three main agents that intervene (teachers, students, peers), and the functionalities through which technology can support the three agents in developing the FA strategies. The teacher, the student's peers, and the student himself or herself are the agents that activate these FA strategies. Through its three *functionalities*, the technology can assist the three agents in developing FA strategies: (1) sending and displaying, which fosters communication among the agents of FA processes (e.g. sending and receiving messages and files, displaying and sharing screens or documents with the whole class; (2) processing and analyzing, which supports the processing and the analysis of the data collected during the lessons (e.g., through the sharing of the statistics of students' answers to polls or questionnaires, the feedback given directly by the technology to the students while taking tests); (3) providing an interactive environment, which creates environments where students can interact to work individually or in groups on tasks or explore mathematical/scientific contents (e.g. through the creation of interactive boards to be shared by the teacher and students, or through the use of specific software that provides an environment in which it is possible to explore). We design a framework in which peer tutoring (PT) is a set of pair relationships in a group graded at different tutoring levels through assigned roles. In PT, one student guides the other in conducting an assignment or learning a concept. In practice, an older student, or someone more experienced, helps a younger or inexperienced student by activating a helping process that accompanies the student in difficulty through various phases of the learning activity: understanding the assignment, exploration of the given problem, bridging to formalization, reflection, and finally evaluation to foster assisted students' awareness of their learning progress. In this strand, we, looking at how a tutor behaves when helping a peer, individuated helping processes or functions that a helper should activate to support a peer's difficulties, identifying them as specific roles. Students are engaged in group peer tutoring, where each student, except the supported student, is required to play a tutoring role at a specific level. We define a graded peer tutoring (GPT) within the group, that is, 'a decomposed form of peer tutoring that takes shape within the group through assigned roles at various levels, which depend on the various stages to prove the statement'. Personifying a peer tutoring role stimulates critical reflection not only at the cognitive level, as it allows students' engagement in the mathematical problem, but also at a metacognitive level, as it fosters students' monitoring skills related to a role to play in the activity. Specifically, students playing a peer-tutoring role are forced to reflect on how one learns, learn strategies on how to learn, and, by receiving continuous feedback from the student who needs to be helped, not only monitor how she or he learns but also improve the awareness of their own learning. In the meantime, the helped students improve learning, and learning to learn. All learn to think mathematically (Schoenfeld, 1992). In our specific activity, considering both the digital component and the FA feedback we want to activate step by step in the helping process, we design three helping roles, depending on the learning phase to be activated.

We face the issue of promoting FA strategies for the development of students' proving competencies in Euclidean geometry by offering them structured tutoring opportunities that allow them to become aware of their own and others' cognitive processes, enabling them to monitor and coordinate them.

RQ: How can digital technology and metacognitive peer tutoring roles activate formative assessment processes, helping secondary school students overcome difficulties in Euclidean geometry proofs or enhance proving competencies? Specifically, how are these factors perceived by students?

Experimental design and definition of roles.

The design of the learning activity foresees helping students face a task requiring proving a statement. Students work in groups of four, structured so that in each group there is a student needing help and

three helpers interacting with her or him in supported by digital technology. According to the GPT model we identify three levels of PT within the group, each corresponding to a specific helping phase that stimulates specific actions T1, T2, T3, and gives in-progress FA feedback (c), (d), (e): T1: exploring and verifying; T2: bridging the gap between informal and formal; and T3: monitoring and managing the entire helping process. Specific, prevalent, but not exclusive, corresponding FA tutoring WT strategies to be activated are: (c) -T1 providing feedback that moves learners forward; (d) -T2: activating students as instructional resources for one another; and (e) -T3 activating students as the owners of their own learning. This tutoring structure is well supported throughout the activity by the FA functionalities (1), (2), and (3) (Cusi et al., 2017) put in motion using digital tools. We associate actions, expected FA strategies and functionalities with specific roles. The Jumper (J) is the protagonist of the formative assessment activity, the receiver of help, to make a cognitive jump that reduces the knowledge gap between him and the other members of the group; the Digital Explorer (DE), an intermediate-level tutor, masterfully uses digital tools, helps to understand the statement and to explore the dynamical configurations of the related diagram through GeoGebra (action T1- strategy (c) - functionality (1); the Bridging Mind (BM), an intermediate-level tutor with a good knowledge and proof techniques, helps to bridge the gap between informal visual reasoning and formal reasoning (action T2 - strategies (c) and (d) - functionality (1) and (2)); the Group Leader (GL) a dual supervisor of the process, intervening to help other colleagues to elaborate the solution, and of the product, and has the skills to do this, high digital skills, excellent logical-deductive skills, and an aptitude for managing (action T3- strategies (c), (d), (e) - functionalities (1), (2), (3)). The assignment of the roles is based on the students' previously ascertained learning state, giving them the opportunity to find the way to stay on track (J), deepen their knowledge based on their awareness of what they already know and how to move on, improve their proving skills (DE, BM), learn how to learn (GL), and receive feedback on their progress. The teacher participates in the activity as an observer of the HGs behind the scenes, intervening to support the group only in case of a block and talking with the GLs with whom she has a privileged communication channel, due to the nature of his /her role.

Methodology.

Implementation of the activity.

The experiment took place in a 10th-grade class of sixteen students in southwestern Italy during the school year 2017–2018, in Classroom 3.0, an environment with a flexible setting: the desks are arranged to form hexagonal islands but, if necessary, they can be broken down and recompose into other configurations. Students were required to work in small groups, specifically four groups named *HG*1, *HG*2, *HG*3, and *HG*4, focused on collectively helping one of the components. In each group, *HGi*, three helpers, *DEi*, *BMi*, *GLi*, and the Jumper *Ji* were identified by the teacher according to their learning state. The Jumper was helped to solve the problem by the other three. The goal of the experiment was to produce proof of the following statement (Figure1), to present it and post it on a Shelf Padlet, a work to be collectively reviewed and evaluated. At the end of the activity, all students were asked to express an overall feedback evaluation of the experience, the analysis of which will give rise to the didactic actions to be undertaken later by the teacher, the second author of this paper.

Draw a circle with diameter AB and centre C and draw the tangent lines in A and B; a third tangent at a point D of the circle intersects the other two at P and Q, respectively. Prove that $PQ \cong PA+QB$.

Figure 1: The statement to prove

Digital tools.

Digital tools supported all phases of the activity. Each *HG* had at their disposal a digital environment consisting of various tools and resources: an island station equipped with tablets connected to Internet and adjacent to traditional boards. All group were equipped with tablets, one for each student, to explore the problem through GeoGebra; boards were used to formalise the proof; and Padlet was

used to send, display and share groups' solutions (GeoGebra and board images) and evaluations of displayed solutions presented by the jumpers; and finally, all students' individual feedback.

Data collection.

All the data concerning the helping-learning activity has been digitally stored on shared Padlet boards. The digital environment contains the shelves where students sent and displayed on the LIM: GeoGebra files of dynamic constructions, images of the boards with the proof of the statement, evaluations of the jumpers' performances, personal feedback of the experience, and a photo gallery.

Data analysis.

Among the collected data we qualitatively analysed evaluations and feedbacks through a systematic and objective identification of some characteristics of FA processes and strategies (identified in the literature) and of the factors triggering them. Specifically, we were looking for signs of the key-FA strategies and functionalities supporting them at the evaluation phase, carried out by the groups, and at the feedback interview on students' individual immediate perceptions of the entire activity, by labelling and classifying sentences according to roles experienced. More in detail, we collected for each Jumper the groups' evaluations and, for each individual role, the impressions of the experience,

Findings and discussion.

To analyse the impact that the designed components have on the activation of students' FA strategies and functionalities, we focus on the evaluations made by groups at the end and on the final interviews.

Evaluation of Jumpers by Helping Groups.

In this phase, the *HG*s evaluate the *J*s's performance. Each group evaluates the other *J*s' performances according to some shared criteria: correctness and completeness of the proof, clarity of the presentation. *Ji* is not evaluated by the *HGi*, having already lived FA moments during the tutoring phases. *GL* coordinates, manages the internal discussion, *processes and analyses* (functionlality (2)) the answers, and *displays* on the Padlet the evaluation expressed through a brief judgement (action T3, functionality (1)). We highlight the peer formative evaluation by looking at the FA signs in the judgments. The argument about *J1* nuances from *not very convincing, quite understandable* to good:

- *HG2*: *J1 argued well* and showed that he understood the problem.
- *HG*3: [...] explained the proof in a way quite *understandable*.
- *HG*4: [...] *J*1's exposition was not very *convincing*, because of the uncertainty about the Theorem to recall, but then an intuitive hint made it clear.

Looking at J3's evaluation, a lack of *self-confidence* appears, but HG4 argues the contrary:

- *HG*1: [...] has a bit of *hesitancy*, exposed the proof of the problem well.
- *HG2*: [...] the proof exposed by J3 was not clear, but *thanks to the help of the group*, she was finally able to prove it correctly using logical deductions.
- *HG*4: [...] the exposure was the most *exhaustive* and *convincing* of all.

According to *HGs*, *J*4's presentation was the least successful; despite this, the evaluation does not highlight the failure but tends to justify it. *HGs* believe that *J*4 needs to understand that he can make the jump with another small effort, that the gap is bridgeable, and that, thanks to this experience, he has already managed to make progress and prove to himself that success is attainable. It emerges the educational value that FA strategies can have on fragile students. Students can assume more responsibility for their own learning and progresses when they are aware of their strengths and areas for improvement. Globally, evaluations *provide feedback that moves learners forward* (c); and

activates *students as the owners of their own learning* (e) through encouraging judgments and emphasising positive performances, with the aim of trying to remove their initial sense of inadequacy and reduce the gap between them and the tutors.

Individual feedback organized by role.

All the feedback shares an appreciation for the use of GeoGebra (functionality (3)) and the organisation in structured PT working groups, and some of them have highlighted other interesting aspects. We begin with significant excerpts from the answers of the Jumpers, the protagonists of the tutoring activity. Signs of *self-formative assessment* take shape when J_1 says he has *become more familiar with the subject*. He recognises his learning improvement, and this is feedback that makes him feel like the *owner of his own learning* (e) and *moves him forward* (c) with a new learning jump:

*J*1: The experience in 3.0 Classroom allowed us cooperative working. It is particularly useful to self-assess and to *become more familiar with the subject*. I believe that these works, aided by digital tools, bring us into contact with the modern world.

*J*2 exhibits on a subject that he has always refused. He recognises the positive role that his comrades have played in the process as well as the validity of the experience in *moving them forward* (c), not only for those who are in difficulty but also for *already capable* students (WT strategies (c), (d), (e)):

*J*2: Effective and interesting experience. It has helped them expose themselves to the subject easily, thanks also to the help of friends. It has helped people with some deficiencies and strengthened some already capable knowledge!

*J*3 seems to suggest structuring the lessons in future like the one just held; both *J*3 e *J*4 emphasise the role of technologies *in providing an interactive environment* (functionality (3)) and from a collaborative perspective and the cooperation to help each other (WT strategy (d)):

- *J*3: The experience we had in the 3.0 Classroom gave us a taste of how the lessons should be carried out. You can take advantage of software like GeoGebra that shows the benefits of dynamic geometry, unlike *static geometry*, in solving a geometric problem. In addition, using online platforms increases collaboration, and joining multiple minds to create a single work will surely lead to an optimal result.
- *J*4: It was a genuinely nice and interesting experience. It allowed me to work better because we worked into groups so that we could compare and help each other, also thanks to the use of tablets and the GeoGebra software.

Let us now look at some feedback from helping roles *DE* and *BM*. *DE*4 emphasises the importance of the learning environment, intended not only as a physical place but also as a digital interactive place. A *DE* who does not have his own tablet appreciates moving around the classroom equipped with a device. In terms of functionality, it refers to *providing an interactive environment* (3) where students can interact to work individually or in groups on tasks or explore mathematical contents (Cusi et al., 2017), and the software *provides feedback moving students forward* (WT strategy (c)):

*DE*4: Unlike other activities that usually take place in the classroom, it allows us to learn encouraging comparison with others and the development of more opinions.

It emerges that, although each had their own means, they all worked with digital technologies:

*BM*1: [...] we put ourselves to prove, but we also made use of new technological tools made available by the school that will certainly help us in the future [...]

With the transition from exploration with GeoGebra to axiomatic *BM3 activates an instructional resource for the Jumper* (WT strategy (d)). *BM3*, attentive to Jumper success, says:

*BM*3: The experience was incredibly positive and productive. Working in a group, each making their own contribution, made me understand that together we can quickly reach a solution. A strong point for me was the drawing conducted with GeoGebra, thanks to which the understanding of the proof was easier and more immediate. I did not find any weaknesses because every group member contributed to the work.

It is interesting to note that cooperation and technology contribute to group and individual growth:

*BM*4: [...] The possibility of cooperating and comparing each other in a constructive way contributes to collective and individual growth.

The Leader, although *engaged* in a tutoring activity (WT strategy (d)), was not bored. He no longer suggests the ordinary lessons because these are more interesting:

GL2: [...] interesting and engaging because it allows us to work in an unusual way from ordinary lessons and to collaborate easily thanks to the availability of the material, pushing us to work better and to attend the lessons with more interest.

GL3 fits the role of leader perfectly, as he gives feedback and moves towards learning to teach, making his colleagues move towards learning. The appreciation of novelty of the method compared to the *usual*, of the *interaction*, of the *comparison with each other* activates FA strategies (d), (e):

GL3: [...] A new way of teaching compared to the usual. We were able to interact more with each other and *better express our opinions* and for me it was also much easier because we worked as a group so we could help each other solve problems and explain them to each other, each with their own ideas.

A cross-cutting objective was to increase the responsibility of leaders towards the community since a knowledgeable student is often used to working alone because he or she believes that others can slow down his pace. GL4 captures this feedback (FA Strategy (d), (e)):

*GL*4: [...] if you are in a group with people, *you are not comfortable with* the activity is counterproductive. However, these activities can be particularly useful to help those who are having difficulty or to *reinforce* concepts that are already known.

Conclusions.

The potential power of formative assessment for enhancing teaching and learning in mathematics education is undiscussed and strengthened using digital technology and metacognitive strategies. An attempt to solve the problem of how to help students overcome difficulties in proving Euclidean geometry can be made by designing an ad hoc activity. We design and implement an activity in which digital technology and roles are the driving forces to activate the FA process and functionalities by structuring groups, according to graded peer tutoring in such a way to produce, at each step, instructions to move forward for the student needing help, supported by digital tools. The first results are promising. Regarding the factors triggering FA processes, the analysis highlights that students appreciate working in structured groups, interacting with each other by working together, and learning about new platforms in a digital technology environment. Regarding the evaluation phase, HGs activated the following processes and strategies: processing and analysing the solution, sent and *displayed* on Padlet, and the presentation; *using an interactive environment*, to express the evaluation; providing through the shared evaluation feedback that moves learners forward; stimulating awareness to be the owners of their own learning. It should be emphasised that students are strongly attached to the 'number' as tend to translate the judgement into a grade, in analogy with what happens within a traditional lesson. This instruction had the objective, on the one hand, to strengthen the responsibility for evaluation for the HG_{s} ; on the other hand, to understand if there is consistency between the numerical grade that could be assigned by the teacher and that attributed by the HGs.

Would the same performance receive a similar evaluation from students and the instructor? This is the pivotal point around which students' dissatisfaction with receiving an evaluation (even a positive one!) revolves and introduces a summative assessment theme to address in a future research direction.

References

Annis, L. F. (2013). The processes and effects of peer tutoring. Human Learning, 10(1), 39-47.

- Baccaglini-Frank, A., & Mariotti, M. (2010). Generating conjectures in dynamic geometry: The maintaining dragging model. *Int. J. of Computers for Mathematical Learning*, 15(3), 225–253.
- Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational* Assessment, Evaluation and Accountability, 21(1), 5-31.
- Boero, P., Garuti, R., Lemut, E., & Mariotti, M. (1996). Challenging the traditional school approach to theorems: A hypothesis about the cognitive unity of theorems. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20th IGPME* (Vol. 2, pp. 121–128). Valencia: PME.
- Cusi, A., Morselli, F., & Sabena, C. (2017). Promoting formative assessment in a connected classroom environment: design and implementation of digital resources. *ZDM Mathematics Education*, Vol. 49(5), 755–767
- Irving, K. I. (2006). The impact of educational technology on student achievement: Assessment of and for learning. *Science Educator*, 15(1), pp. 13-20.
- Moore, R. C. (1994). Making the transition to formal proof. Ed. Studies in Mathematics, 27, 249-266.
- Moreneo, C., & Duran, D. (2002). Frameworks: Cooperative and collaborative methods. Edebe.
- Olivero, F., & Robutti, O. (2007). Measuring in dynamic geometry environments as a tool for conjecturing and proving. *Int. Journal of Computers for Mathematical Learning*, 12(2), 135–156.
- Raman, M. (2003). Key ideas: What are they and how can they help us understand how people view proof? *Educational Studies in Mathematics*, 52, 319–325.
- Roschelle, J., & Pea, R. (2002). A walk on the WILD side: How wireless handhelds may change computer-supported collaborative learning. *Int. J. of Cognition and Technology*, 1(1), 145-168.
- Sadler, D. R. (1998). Formative assessment: Revisiting the territory. Assessment in Education: principles, policy & practice, 5(1), 77-84.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.
- Swan, M. & Burkhardt, H. (2014). Lesson design for formative assessment. Ed Designer, 2(7), 1-24.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.
- Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*. 48(1), 101–119.
- Wiliam, D., & Thompson, M. (2007). Integrating assessment with instruction: What will it take to make it work? In C. A. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning* (pp. 53–82). Mahwah, NJ: Erlbaum.
- Zazkis, D., Weber, K., & Mejía-Ramos, J. P. (2016). Bridging the gap between graphical arguments and verbal-symbolic proofs in a real analysis context. *Educational Studies in Math*, 93, 155–173.