Designing with the Teaching for Robust Understanding framework: indicators for the activation and realization of formative assessment strategies

Alessandra Boscolo[1], Francesca Morselli[2], Simone Quartara[3] and Elisabetta Robotti[4]

[1]DIMA University of Genova, Italy; boscolo@dima.unige.it

[2] DIMA University of Genova, Italy; morselli@dima.unige,it

[3]IIS Italo Calvino, Genova, Italy; simone.quartara@calvino.edu.it

[4] DIMA University of Genova, Italy; robotti@dima.unige.it

The study explores the implementation of formative assessment strategies in the context of algebraic thinking and argumentation within a teaching experiment. Teaching for Robust Understanding framework and theoretical references guide the task design. Specific indicators for two formative assessment strategies are developed, tailored on the learning goals, and examples of their instances (activation and realization by teachers and students) are provided. Future work will extend this analysis to other strategies and assess its applicability in other learning sequences.

Keywords: Formative assessment, TRU framework, community of inquiry

Introduction and background

Our contribution addresses task design and the assessment of algebraic thinking and argumentation as key learning objectives. We rely on Black and Wiliam (2009)'s characterization of formative assessment as a method of teaching where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited". (p. 7). Schildkamp and colleagues (2020) underscore the challenges teachers face in implementing formative assessment effectively within their classrooms. In their literature review, they point out that it is imperative to integrate formative assessment seamlessly into the teaching and learning process, surpassing the mere addition of formative assessment activities. Moreover, teachers should be inclined to share the responsibility of instruction with students, thereby renegotiating the role and authority of the teacher in the teaching and learning process. Teacher prerequisites supporting this shift in perspective and practice encompass pedagogical content knowledge (essential for identifying student difficulties and offering feedback), the ability to articulate and share learning goals with students, and the capacity to facilitate class discussions. Additionally, social factors, such as collaboration with colleagues and cultivating positive relationships with students, play pivotal roles. All these factors underscore the importance of making teachers able to effectively implement formative assessment in their daily classroom practices, recognizing the complexity inherent in the teaching and learning process. We contend that creating an appropriate context for teachers to reflect on their practice, share and compare experiences, and providing them with theoretical tools supporting the design and implementation of teaching sequences, including formative assessment activities as integral components, is paramount. Our study was conducted within the community of inquiry DIVA (Didattica, Inclusione, Valutazione formativa, Argomentazione – Didactics, Inclusion, Formative Assessment, Argumentation), established at the Mathematics Department of the University of Genoa in February 2023. This community of teacher-researchers has been collaborating to identify theoretical tools for reflection, address specific needs and areas of interest, and design teaching and learning sequences. The initial theoretical tool shared and utilized was Schoenfeld's TRU framework (Teaching for Robust Understanding) (2016), that identifies five dimensions for learning: mathematics, cognitive demand, equal access to content, agency, ownership and identity and formative assessment. The mathematical dimension is at the core of the model and the other dimensions are shaped around it. The dimensions do not contain prescriptive "recipes" for teachers but rather guidelines for creating powerful learning environments, that result in students becoming resourceful thinkers and learners. The dimensions provide an analytical tool for the observation and reflection on one's own teaching practice and can be used for designing, evaluating the effectiveness of the intervention and thinking about the next steps in teaching action. In this contribution, we present a teaching and learning sequence conceived and implemented within the TRU framework and we study to what extent formative assessment strategies were implemented. Wiliam and Thompson (2007) discuss five key strategies that may help promoting formative assessment in the classroom: FA1) clarifying and sharing learning intentions and criteria for success; FA2) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding; FA3) providing feedback that moves learners forward; FA4) activating students as instructional resources for one another; FA5) activating students as the owners of their own learning. Not only the teacher, but also the peers and the student himself/herself may act as agents of formative assessment.

Research Design

Our exploratory study is based on a teaching experiment, involving a teacher (the author SQ) who is a member of the DIVA community of inquiry. Despite being an experienced teacher, who already took part to teaching experiments concerning formative assessment (Morselli &

Quartara, 2023), this represents his first attempt to design a learning activity through the framework TRU. The activity took place in a grade 9 class (18 students) of an upper secondary school with a scientific orientation.

In reference to the mathematical content dimension, which is at the core of the TRU framework, the activity at issue was aimed at the development of algebraic thinking and of argumentative competence, with a strong focus on the interaction between them. As mentioned earlier, what sets DIVA apart is its approach to design, guided by the theoretical tool TRU, and the practice of sharing explicit reflections with teachers in the community. These reflections are often guided by additional theoretical tools that cater to the various dimensions of TRU and the specific content that their proposals intend to cover. In this case, additional theoretical tools refer to algebraic thinking and argumentation. Algebraic thinking is explicitly linked to Arcavi's conceptualization of symbol sense (1994). The development of symbol sense involves: understanding how and when to use symbols to represent relationships, generalizations, and proofs; being aware that in some cases it is more convenient to abandon symbols in favor of other approaches; dealing with the dialectic between manipulation and interpretation of symbols; being aware of the possibility of creating symbolic expressions, and being able to create them; being able to select, but also to abandon or change a symbolic representation; being aware of the need to constantly check symbol meanings during problem-solving; being aware of the fact that symbols may play different roles. Concerning the second objective, that is, the development of argumentative competence, we refer to Habermas's characterization of "rational behavior" (Morselli & Boero, 2010), thus identifying three components: epistemic (inherent in the correctness of the argumentative process); teleological (inherent in the problem-solving character of the process, and in the related strategic choices); communicative (related to the comprehensibility and communicative choices of the argumentation). All the specific theoretical references were shared with the teacher before starting the design and implementation. Once fixed the mathematics dimension, the design of the activity was structured to take into account the other dimensions of the TRU framework. Due to space constraints, we summarize in Figure 1 the structure of the activity, involving 4 stages, specifying the TRU dimensions motivating the introduction of each stage. We will defer this description to future work. The activity is based on the resolution of an algebraic item selected from the INVALSI national assessment repository GESTINV[1] (D14 G10 year 2010). In the first individual stage (10 minutes), students were asked to explore and conjecture around the following open-ended question: "If n is any natural number, what do you get by adding the

[1] https://www.gestinv.it/Index.aspx

three numbers 2n+1, 2n+3, 2n+5?". Consequently, they were asked to conjecture the truth value of the following statements: Mario's one ("You always get the triple of one of the three numbers"); Luisa's one ("You always get an odd number"); Giovanni's one ("You always get a multiple of 3"). In the second stage, students were asked to discuss in small homogeneous groups and compare their own conjectures, formulated in the previous stage, with group members, answering the following multiple-choice question: "Who is right? a. All of them, b. Only Mario, c. Only Luisa, d. Only Giovanni." Students were required to come to a consensus on a solution and produce a written argument in which their solution is accompanied by a justification regarding the truth value of each of Mario, Luisa and Giovanni's statements. In the third stage, involving a whole class discussion, the teacher displayed the students' responses on the whiteboard and asked each group to narrate the solutions and arguments they previously developed, involving the students from other groups as well to ask, comment, and compare strategies. During the discussion, aspects related to the algebraic correctness of the solutions were examined, along with the formulation of arguments, and the role that algebra had played in the diverse solutions and argumentations. Finally, the students were asked to complete a self-assessment questionnaire, with a Google module, containing questions aimed at monitoring the aspects of the five TRU framework dimensions on which the design focused. All the discussions were video-taped and transcribed. All the students' productions were collected.

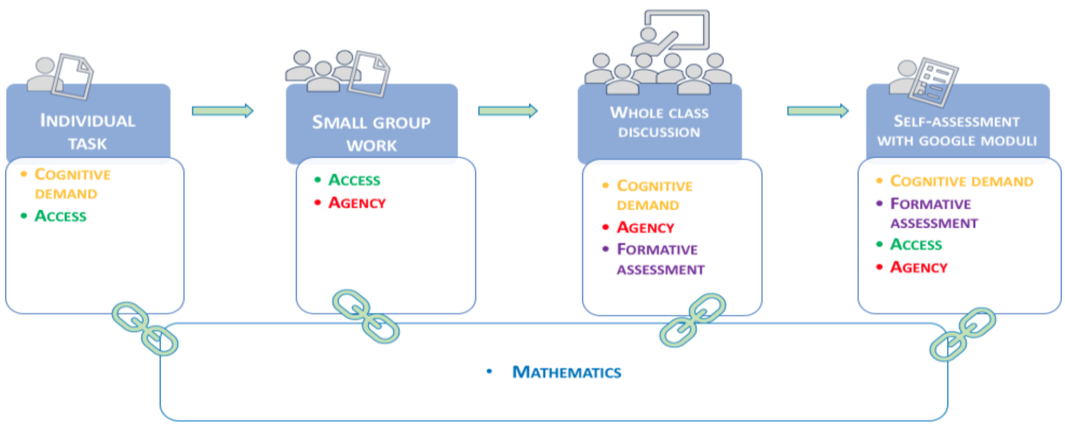

Figure 1: Stages of the activity

Analytical tool: the indicators

To study the actual implementation of formative assessment strategies (Wiliam and Thompson, 2007), we developed specific indicators to detect the *activation* and *realization* of each of them, declined in the specific case of algebraic thinking and argumentation, and indicators for the effective activation of the strategy. The indicators were theoretically set up by tailoring formative assessment strategies on the specific learning goals of the activity, guided by the theoretical frameworks used as references: symbol sense and rational behavior. Due to space constraints, here we present indicators for FA1 (*clarifying and sharing learning intentions and criteria for success*) and FA2 (*engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding*). These strategies were selected for being the most related to the design of the sequence and to the theoretical frameworks used to frame the learning goals. The strategies FA1 and FA2 were categorized based on the specific learning goals they addressed: FA1.1 and FA2.1 pertain to algebraic thinking, while FA1.2 and FA2.2 are associated with rational behavior.

Table 1: Indicators for FA1 and FA2

	Indicators for activation	**Indicators for realization**
FA1.1	a. the teacher underlines/makes explicit the importance of using symbols to represent relationships, generalizations and proofs b. the teacher points out that in some cases it is more convenient to abandon symbols in favor of other approaches c. the teacher makes explicit the importance of dealing with the dialectic between manipulation and interpretation of symbols d. the teacher underlines the possibility of creating symbolic expressions, and being able to create them; e. the teacher underlines the importance of selecting but also abandoning or changing a symbolic representation f. the teacher underlines the importance of constantly checking symbol meanings during problem solving g. the teacher underlines that symbols may play different roles	The student shows to be aware of the learning goals and criteria for success concerning algebraic thinking (e.g. mentioning the importance of using algebra to generalize)
FA1.2	a. the teacher underlines the importance of providing explanations b. the teacher clarifies the criteria for a good argumentation c. the teacher promotes a reflection on the epistemic component (correctness) d. the teacher promotes a reflection on the teleologic component (strategy to solve the problem, goal-oriented actions…) e. the teacher promotes a reflection on the communicative component (comprehensibility of the solution, …) f. the teacher promotes a reflection on the role of examples in argumentation	The student shows to be aware of the learning goals and criteria for success concerning argumentation (e.g. recognizing the need to move beyond numeric examples in proving)
FA2.1	The design encompasses activities such as small group work/class discussion/self assessment, aimed at: a. comparing solving strategies and solutions b. reflecting on strengths and weaknesses of the solving strategies /e.g. choice of the formalization) c. making students explicit their solving process	The teacher poses questions aimed at eliciting evidence of student understanding, with reference to algebraic thinking The student provides evidence of his/her understanding, with reference to algebraic thinking.

FA2.2	The design encompasses activities such as small group work/class discussion/self assessment, aimed at: a. comparing argumentations b. reflecting on strengths and weaknesses of the proposed argumentations (e.g. role for the numerical examples in proving) c. making students explicit their choices related to argumentation (e.g. use of specific terms)	The teacher poses questions aimed at eliciting evidence of student understanding and realization of argumentation. The student provides evidence of his/her understanding, with reference to argumentation.

Discussion of results

We employ the aforementioned indicators to detect *activation* and *realization* of formative assessment strategies, in the teaching and learning sequence. Due to space limitations, we present only two examples of this analysis. For strategy FA1 (*clarifying and sharing learning intentions and criteria for success*), we can identify instances of *activation* in the teacher's efforts to clarify the desired learning objectives. The actual *realization* of FA1 can be found in the students' protocols, their contributions during discussions, and answers in the self-assessment questionnaires, reflecting their grasp of the learning objectives. The example we present specifically pertains to FA1.1. An instance of the *activation* of FA1.1, particularly with reference to indicator *a*, can be seen in the following excerpt (discussion, min. 26.28):

Teacher (T): Well, it's the power of algebra: the ability to generalize and condense infinite numbers into a single symbolic representation. Indeed, that's how it is. 2n: we've written down all those infinite even numbers in a single expression.

Examples of the *realization* of FA1.1, related to indicator *a*, were observed both during the class discussion and in students' responses to self-assessment questionnaires. In the class discussion, students justified their solutions by highlighting the role played by algebra, prompted by the teacher, as shown in the following excerpt (discussion, min. 02.49 - 03.30):

T: How did we go from 6n+9 to 3(2n+3)?
Mary: We performed factorization
T: What was its purpose?
Mary: To find a number, which is 3k. This factorization became k.
T: Matt, would you like to help Mary?
Matt: We modeled what was inside the parentheses by naming it k, and then we got 3k, which means - well, 3k represents all the multiples of 3.

Further, in the self-assessment questionnaires, answering the question “Where does algebra come into play, and how has it helped you?”, students provided responses like the following: "It helped me generalize parts that would have otherwise required infinite examples." (Elia).

Looking at strategy FA2 (*engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding*), we provide an example of its *activation* for FA2.2, involving all three indicators (*a*, *b*, *c*): the strategy focused on comparing protocols with group responses during collective discussions. It allowed the teacher to address a critical aspect during discussions, aiding students in transitioning from arithmetic to algebra: the role of numerical examples in justifying their solutions to the task. An example of the *teacher's realization* of FA2.2, specifically indicator *b*, can be seen in the teacher's interventions during the whole class discussion (min. 24.12), seeking to understand the challenges arising from students' arguments, especially by providing feedback on where to focus attention, particularly regarding the role of numerical examples:

T:	Alright. Why? Florin, if you remember, Florin, why did you provide an example??

Realization of FA2.2 (indicator *b*) concerns both the teacher, highlighting the goal indicated in FA1.2 *f*, and student intervention related to understanding, as referenced in FA1.2 *f*, can be found in the following excerpts of the discussion (min. 07.35 - 08.02):

T:	Do you agree that just one example, for instance, a specific example like they took n=0, which is particular as the smallest number, is enough to prove that Luisa is correct, that is, you always get an odd number?
Phil:	Since you can insert infinite numbers, well, it's not enough. But one counterexample is enough to refute the theory.

While showcasing these examples, it's important to note that they represent only exemplar instances of the *activation* and *realization* of the formative assessment strategies, concerning exclusively some specific indicators. In our analysis, we observed that these strategies were activated on several occasions and, further than by the teacher, by a multiplicity of students in the class. This provides a measure of the effective implementation of formative assessment concerning FA1 and FA2 in the teaching and learning sequence, in term of pervasiveness.

Conclusions, limitations and further directions

In the present contribution, we have shown how designing a teaching and learning sequence within the TRU framework, where all the dimensions are shaped around the mathematical learning goals, complemented by specific theoretical references (symbol sense and rational behavior), allows us to formulate a tool for analysis and, consequently, to evaluate the effective implementation of formative assessment strategies (Wiliam & Thompson, 2007) in the classroom. Indeed, the study leads us to conclude that the modes of designing the sequence allowed us, firstly, to identify specific indicators to monitor the activation and realization of formative assessment strategies. Secondly, from an initial analysis using the formulated indicators, as shown in the examples illustrated, we could assess that the strategies FA1 (*clarifying and sharing learning intentions and criteria for success*) and FA2 (*engineering effective classroom discussions and other learning tasks that elicit evidence of student* *understanding*) were realized in the classroom. Although, due to space constraints, we were able to show traces of the activation regarding only some indicators and with few examples of realization taken from the discussion and self-assessment questionnaire, the comprehensive analysis highlighted the pervasiveness of formative assessment strategies (strategies were activated in several occasions and different students were involved). In further work, we will illustrate our analysis in relation to the other strategies. A further direction involves generalizing the analytical tool, thus studying whether the development of contextual indicators for evaluating the effective implementation of formative assessment strategies can be transferred to the analysis of other teaching activities with specific learning goals.

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