Elaborated feedback for interpreting graphs in Secondary Education

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Interpreting and extracting information from graphs can be challenging for secondary education students. While feedback often yields positive effects in correcting student errors, there remains a gap in understanding how prior knowledge influences performance in the context of functions and their reactions to feedback. In this study, 68 students solved a task with a graph through an e-assessment tool (STACK) that allows multiple opportunities to solve it and progressively provides feedback for arriving at the solution. Our results reveal that most medium achievers made a standard error and overcame it after receiving one or two hints, whereas low achievers committed uncommon mistakes and required at least three hints. Our study also shows different reactions to feedback; some low achievers felt overwhelmed when connecting the feedback provided across multiple attempts as the information was presented individually.

Keywords: Feedback, functions, graphs, STACK

Graphs and feedback through e-assessment

Dealing with graphs is an essential skill for secondary education students to tackle real-life situations and advance in higher training stages (Planinic et al., 2012). Previous research has revealed student difficulties interpreting graphs (Graham & Sharp, 1999; Planinic et al., 2012; Ortiz-Laso, 2017). One of the main difficulties is recognizing which graph features should be used to extract information (Graham & Sharp, 1999). In this line, Ruchniewicz and Barzel (2019) outlined that electronic assessment (e-assessment) tools are helpful for students to reflect on graph interpretation and attain the necessary knowledge to respond to a mathematical task.

This task-related knowledge can be delivered through diverse feedback varying from simple to elaborated. Simple feedback relates to how well a task has been performed (Narciss et al., 2022) and comprises three types: knowledge of result (KR), knowledge of performance (KP), and knowledge of correct result (KCR; Narciss, 2008). KR provides information about response correctness (e.g., correct or incorrect), KP gives the number of correct responses, and KCR delivers the correct task solution. Elaborated feedback provides concise information and can be divided into five types: knowledge on task constraints (KTC), knowledge about concepts (KC), knowledge about mistakes (KM), knowledge on how to proceed (KH), and knowledge on metacognition (KMC) (Narciss, 2008). KTC clarifies task nature, subtasks, processing rules, and requirements (e.g., "The first step of the correct solution would be…" Pinkernell et al., 2020, p. 223). KC delivers conceptual information to reach the solution (e.g., providing a mathematical definition), whereas KM provides mistakes' location, type and origins (e.g., "You probably made this error..."; Pinkernell et al., 2020, p. 223). KH guides the responder into the right solution, correcting specific mistakes and providing hints and examples (e.g., "Do not ignore the cards that are negative instances of the given concept, as they provide useful information"; Narciss, 2013, p. 19). Finally, KMC offers guiding questions attracting attention to metacognitive strategies (Narciss et al., 2022).

Previous studies on elaborated feedback have investigated its usefulness according to students' previous knowledge. Fyfe and Rittle-Johnson (2016) stated that both high and low achievers benefit from elaborated feedback, whereas Pinkernell et al. (2020) discovered that only low achievers improve. The type of elaborated feedback also influences its effectiveness; Pinkernell et al. (2020) found that among German low-achieving secondary education learners, the benefits derived from KM or KTC surpassed those obtained from KH.

Research question and methods

To shed light on the effectiveness of elaborated feedback, this study investigated how secondary education students request and react to feedback provided through an e-assessment tool and how it influences attaining the correct solution. A sample of 68 students from two high schools in Cantabria, Spain, was selected. Students were in the third and fourth grades of compulsory secondary education, aged 14 to 16. They received instruction through textbooks on functions and graphs and represented three achieving groups: low, medium, and high.

Proposed task and designed feedback

To assess our objectives, we adopted Ortiz-Laso's (2017) graph task (Figure 1) in which student responses were classified into three sets: correct answers (100 minutes), expected error (120 minutes; students did not realize that Juan was not moving between minutes 50-70), and unexpected errors (non-typical errors related to students' lack of knowledge on graphs).

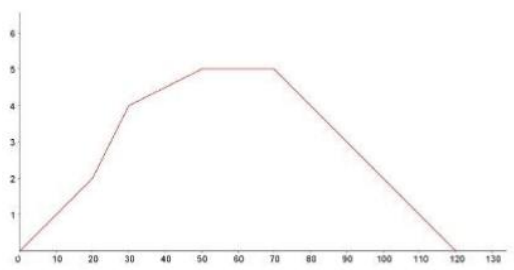

Figure 1: Juan's distance from home

Juan leaves home to exercise in a mountain zone. He starts walking at his usual pace and then alternates between running and walking at different paces. The graph in Figure represents his activity, where the x-axis is time (in minutes) and the y-axis is the distance from home (in kilometers). How much time does Juan spend in motion?

To deliver elaborated feedback on the errors classified by Ortiz-Laso, we considered the following three Narciss' (2008) categories (KR, KH, and KC) to be provided through STACK (System for Teaching and Assessment using a Computer Algebra Kernel). This tutoring system was chosen because it dispenses specific feedback for each error and allows tracking of every response (Sangwin, 2013). KR feedback was designed for correct answers, whereas KH and KC were devised for the wrong ones (Figure 2). For the expected error (120 minutes), the solvers received automatic 'Knowledge on How to proceed' to reflect on each graph part (A: *Is Juan not moving at any moment?*). Then, they also could request progressive hints as follows: extra KH (A1: *What does it mean for the graph to have a horizontal part?*), KC related to the constant part of the graph (A2: *The graph indicates the distance between Juan and his home. If the graph is constant during a period, it means that Juan is not moving during that time.*), and further KC incorporating an explanation about the time variable (A3: *Remember that the walk lasted for 120 minutes, and Juan was not moving between minutes 50-70*). For unexpected errors, the solvers got automatic KC that included a graph description (B: *The graph shows the distance between Juan and his home at each moment. His route finishes when the distance is 0 again*). Finally, they also had the opportunity to request KC and KH in to get a task reformulation (B1: *The graph shows Juan was outside for 120 minutes. You are asked how many of those minutes he was moving*).

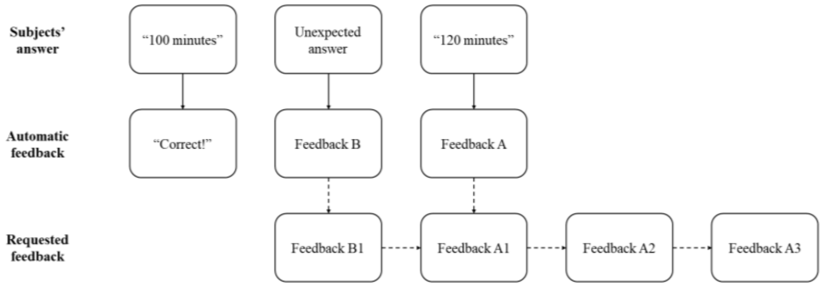

Figure 2: Offered Feedback

For data collection, the dataset generated in STACK for each student attempt and cognitive interviews were employed. Although a mixed-method approach was applied, data analysis was qualitative in nature. After classifying students' responses under previously described categories, and the number of attempts, students were asked about the reasons behind their responses and the way in which they reacted to feedback.

Results and discussion

The analysis revealed that about one-third of students achieved the correct solution in the first attempt. The remaining ones failed on the first attempt, evidencing difficulties related to the interpretation of graphs, as reported by Graham and Sharp (1999) and Planinic et al. (2012). The first answer varied according to students' academic achievement; the medium achievers normally made the expected error '120 minutes', while the low achievers generally provided a set of unexpected responses. The cognitive interviews revealed that the latter responses stemmed from a lack of students' skills to interpret functions; for example, during a cognitive interview, one of the low achievers stated: "I was unsure about what to reply because I didn't quite understand the graph". Those who replied 120 minutes in the first attempt understood the task context, but they either interpreted the graph globally or did not comprehend the meaning of having scope 0. In both cases, they provided the correct answer after receiving the first feedback. During the cognitive interviews, one of the students stressed: "After reading the feedback, I realized that I needed to look at every part of the function […] The solution was not the biggest value reached by the function in the x-axis".

Differences were also observed in how the students reacted to the feedback. Medium achievers tended to be reluctant to ask for feedback, and instead, they reattempted the task. About half of these students got the correct answer after receiving the automatic feedback (A), while the others required an extra hint (A1). In contrast, low achievers were willing to demand extra information before providing a new answer. The ones who got the correct answer achieved it after receiving two extra hints. The others did not request a third extra hint despite needing to arrive at the correct answer. They reported feeling overwhelmed and frustrated because the hints did not appear simultaneously, having to retain information from previous ones. One student claimed: "Some hints were difficult to understand without thinking about the previous ones”. In this case, the students were not able to solve the task, but they started to think about their own work, being conscious of the need to engage in learning, something already observed by Ruchniewicz and Barzel (2019). The above suggests that when designing feedback, it should be both concise and presented in an accumulative way, at least for low achievers. Our results should be interpreted with caution as the study involved a reduced number of students tackling a unique task. Further investigations into learning graphs and e-assessments are thus necessary to support these findings.

Acknowledgment

This work is supported by the Erasmus+ Programme of the European Union (2021-1-DE01-KA220-HED-000032031; https://www.authomath.org/). It reflects only the authors’ view, and the European Commission’s support does not imply endorsement of the content and is not responsible for its use.

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