Development and validation of a three-dimensional framework for the classification of authentic tasks in mathematics

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Global trends in mathematics education place increasing importance on problem-solving skills in authentic contexts. In this paper, the authors propose a three-dimensional framework for the classification of authentic tasks in mathematics. These dimensions, which combine the complexity of the task but can at the same time be analyzed separately, are (1) complexity of the mathematical model, (2) context of the given problem, and (3) strategic complexity connecting the problem and the mathematical model. To validate the framework, the levels of complexity for 12 tasks are determined and students' performance on these tasks is compared. This framework is a valuable tool for designing both learning and assessment tasks.

Keywords: Authentic task, Mathematical problem, Assessment of complexity.

Introduction

Global trends in learning goals shape the International Student Assessment Programme (PISA). PISA's latest mathematics framework highlights the importance of mathematics in today's changing world driven by new technologies. After graduation, citizens are expected to be creative and engaged, making non-routine judgments (OECD, 2023a). This means moving beyond the way mathematics is traditionally taught and prioritizing mathematics learning based on real-life examples (Kaiser & Schwarz 2010). The latest PISA results show that Latvian students perform above average in mathematics at the lower proficiency level, but below average at the higher proficiency level (OECD, 2023b). This indicates that Latvian students need to improve their performance on problem-solving tasks in which they must think without a pre-known algorithm and in which several solutions are possible, so that they need to be more creative and evaluate their ideas. In this situation, national tests are not a driving force either. Previous analyses of Latvian national assessments show a lack of tasks with authentic contexts and show poor indicators of higher-order thinking skills. This study aims to develop and validate a multifunctional framework for designing mathematical problems with authentic contexts at different levels of complexity. A framework would help to build a common understanding of what characterizes higher-level problems, in order to promote the development of higher-order thinking skills among students.

Literature review

There is no common interpretation of what is considered an authentic task or an authentic context. Some authors define it not as a property of the problem, but as a property of the connection between the problem and its solver (Kramarski, Mevarech & Arami, 2002). From this perspective, the same problem will be authentic for some students and not for others. Others stress that authenticity is determined by the fact that problems are purposeful and meaningful (Jurdak, 2006). In this study we assume that tasks in authentic contexts "require a 'real-world' element whether in terms of meaningfulness, relevance and/or application to the personal lifeworlds of learners, as well as an element of connectedness to other subject domains and contexts beyond the textbook and school" (Tan & Nie, 2015, p 22).

When designing mathematical tasks in authentic contexts, it is important to take into account that their content consists of multiple dimensions. Dimensions are connected in the task but can also be isolated to be analyzed separately. Pugalee and colleagues (2002) identify four dimensions: thinking and reasoning, discourse, mathematical tools, and attitudes and dispositions. Paredes and colleagues (2020) point out three main aspects that should be considered when classifying mathematics tasks: (1) the context in which the task is placed, (2) the variety of responses to the task, and (3) the level of cognitive demand activated when solving the task. Maaß (2010) has studied previously created classification versions and introduced a new, highly detailed scheme for the classification of mathematical modeling tasks. It categorize tasks based on their characteristics and specific elements. Not all these elements affect complexity of the task. To create assessment tasks, a framework is needed that outlines how complexity increases. It is crucial for developing an accurate assessment tool to mark the direction of intervention and improve both teaching and learning.

Methods

In this study three dimensions are distinguished which determine how complex a task is: (1) context of the given problem, (2) complexity of the mathematical model, (3) strategic complexity connecting the problem and the mathematical model (Table 1). According to the PISA 2003 Mathematics framework, each situation is more or less related to the student's world (OECD, 2003). This transfer distance forms the first dimension. The second dimension is the complexity of the mathematical model. The Structure of the Observed Learning Outcome (SOLO) taxonomy's (Biggs & Collis, 1982) unistructural, multistructural and relational levels are the basis for defining this dimension. The third dimension is about the relationship between a given situation and a mathematical model, or the ability to formulate, interpret and evaluate (OECD, 2023a).

Table 1: Three-dimensional framework for a classification of authentic tasks in mathematics

	Level 1	Level 2	Level 3
Context of the given problem	A simple, straightforward, familiar situation, often in a personal context.	The situation is described using several sources of information. Although the situation is relatively familiar, it requires a deeper understanding of the context.	Complex, relatively new situation. Situation analysis or generalization is needed
Complexity of mathematical model	A simple mathematical model consisting of a single content element.	Multiple unrelated elements, an algorithm, a learned procedure.	Multiple related elements, requiring a deep understanding of mathematical concepts.
Strategic complexity connecting the problem and the mathematical model	There is a clear solution path, which may be explicitly or implicitly given in the instructions for the task. The problem allows for one correct answer.	A solution path may be chosen. There is a need to justify/explain the answer as the context allows interpretations. Assumptions need to be made.	The limitations of the context must be considered, assumptions must be made and the relevance of the mathematical model to the problem must be evaluated. The solution to the situation may differ significantly depending on the mathematical model chosen.

This study is a first validation step to test whether the complexity of the tasks created by the framework increases. The created tasks are part of the pilot study for the national numeracy monitoring in grades 6 and 7. A total of 856 participants took part in the study. The pilot study was conducted using three different item sets. The total number of items is 27, of which 6 are anchor items, identical in all tests. Items were coded based on their mathematics topic – A stands for “ratios and relationships”, G-“geometry”, L-“time and speed”, E-anchor items. The following numbers represent the task number in the student's worksheets. To ensure reliability, coefficient Cronbach's alpha was calculated for each set of results. Tasks with an authentic content were selected by experts according to the following criteria: (1) match at least the first level of the framework in each dimension, (2) fit the Rasch model. 12 tasks were selected from three item sets for the study. The Wright maps were analyzed comparing the position of different level items against the anchor items.

Results

The calculated Cronbach's alpha coefficients are 0,76; 0,67; 0,76. Considering that this is the initial pilot study, we consider these Cronbach's alpha coefficients to be acceptable to ensure reliability. In Figure 1 all the items selected for the study are framed and the determined complexity level is shown. For example, 1/2/3 means level 1 in context dimension, level 2 complexity of the mathematical model, and level 3 strategic complexity.

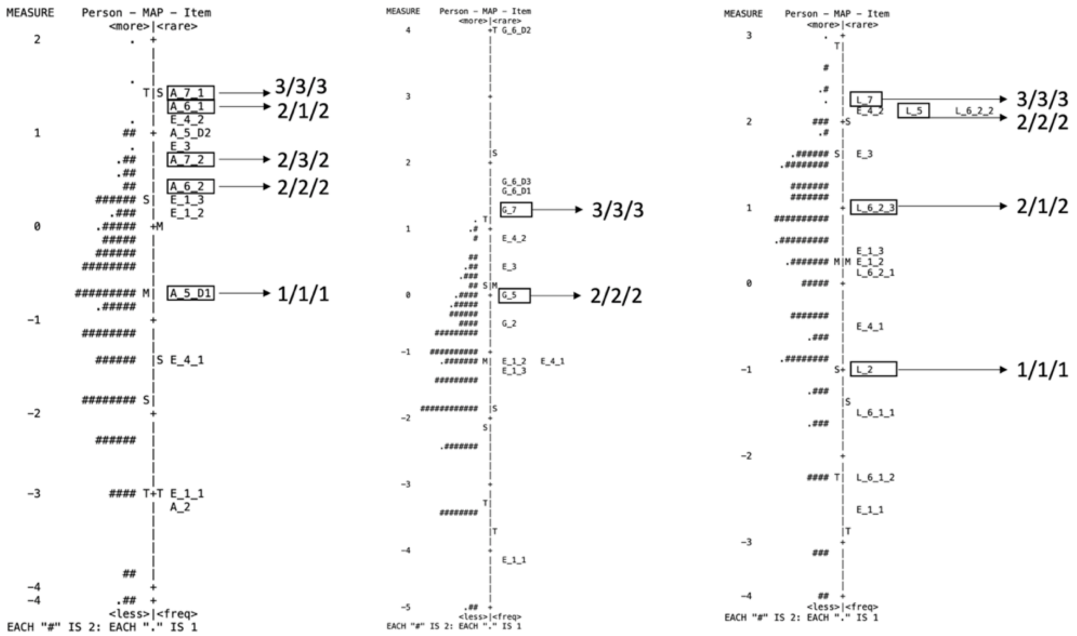

Figure 1. Item positioning on Wright maps of three item sets.

If looking at each set and each dimension separately, tasks with a higher level of complexity are positioned higher in the Wright maps, indicating that students' performance decreases with increasing levels of complexity. Item A_6_1 within the first item set does not fit the expected hierarchy in the second dimension – complexity of the mathematical model. This task requires calculating the unknown term of a proportion, which is in the curriculum at exactly the time the test is taken. Some students may have learned this skill, so it could used as a learned algorithm, but some students made up the solution in the given context.

Conclusions

The three-dimensional framework for a classification of authentic tasks in mathematics allows to purposefully increase the level of complexity. It is important to have a step-by-step approach in learning, but it is also essential in assessment to design tasks so that their complexity increases gradually to enable as many students as possible to demonstrate their best performance. It is crucial to further develop and implement the framework.

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