Students' feedback process using automated post-submission report in their modeling process

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In this study, we explored a pair of students' interactions with the automated post-submission report to learn how they use this tool to deepen their own modeling process. The students worked on a modeling activity consisting of four example-eliciting tasks in which they were asked to construct position-over-time graphs. The findings show that by interacting with the post-submission report, new sequences of mathematical modeling competencies use were identified, which enables fostering the student's interpreting and validating the mathematical results.

Keywords: Feedback process, mathematical modeling, modeling process.

INTRODUCTION AND THEORETICAL BACKGROUND

Feedback is an ongoing process in which learners make sense of information related to the task or process of learning, which is provided by an agent (e.g., teacher, peer, book) regarding aspects of one's performance or understanding (Hattie & Timperley, 2007), to fill a gap between what is understood and what is aimed to be understood (Sadler, 1989). Indeed, studies have shown that the feedback given to students has a greater impact on their achievement than any other teaching strategy (Hattie & Timperley, 2007). In their study, Yerushalmy et al. (2023) discussed how artifacts, particularly personal feedback that are designed as tools by others, might become instruments for learning. They referred to personal feedback as a mirror that describes an instance by offering a task designer's description of a learner's example. This description includes the characterization of students' examples in words.

Using technology in mathematics education affects the teaching and learning processes; specifically, it can be used to support the feedback process in many ways. Technology can be used to generate outputs such as numeric grades, written reports, and statistics for use by teachers (Sangwin & Köcher, 2016), students (Yerushalmy et al., 2023), or both (Abu Raya & Olsher, 2021; Sadler, 1989). Some technologies also have the feature of reporting to the teacher regarding the tools that students use when they solve the task (Abu Raya & Olsher, 2021), while others have the feature of analyzing the answers according to mathematical characteristics (Abu Raya & Olsher, 2021). In addition, the feedback process can be supported by the effective use of digital tools, such as simulations and interactive diagrams, which provide learners with immediate information that can help them solve problems and can positively affect various competences, such as understanding, validating, and interpreting, which are considered modeling sub-competences (Greefrath, 2011). Thus, technology can promote mathematical modeling, which is an important competency that students of all ages ought to acquire, owing to its role as a method for better understanding the world around us. Greefrath (2011) described two areas in which the modeling process occurs: the rest of the world and mathematics, and introduced a modeling cycle (MC) that represents key phases (in black font) and transitions between them (in red font) in the processing of reality-related problems. He suggests that digital tools may be useful at each step of this process (Figure 1). For example, for experimenting, by transforming a real situation into a geometrical model, with the help of dynamic geometry software.

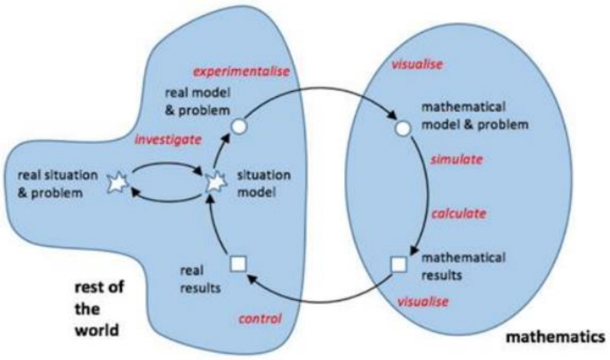

Figure 1: Modeling cycle with the added influence of digital tools (Greefrath, 2011)

Mathematical modeling literature highlights the need for more research to better understand how technology may assess or foster mathematical modeling competencies (MMC) (Cevikbas et al., 2022). Therefore, the purpose of this study is to gain insights into automated post-submission reports serving as a bridging tool between the literacy phenomenon and the given mathematical model. We asked: (1) How do students use post-submission reports while working on the modeling activity? (2) Do students' feedback processes using automated post-submission reports affect their MMC?

METHODOLOGY

Population

We present a case study involving a pair of 9th graders (14-year-old students), Ray and Lour (fictitious names), who completed a computer-based modeling activity with their teacher during mathematics lessons. The pair of students was chosen based on their educational level (medium-high), their good expressiveness, and their seriousness.

Tools

The research tools included a modeling activity designed on the STEP platform and class observations. The STEP platform was designed to support the assessment of various patterns of open-ended example-eliciting tasks. The students' submitted examples were analyzed online according to specified characteristics, producing a post-submission report for the students (Harel et al., 2019). These characteristics can provide information that goes beyond whether students' submissions are correct or not, can give students a sense of the breadth of their personal example space, and may be resources to create a shift in students' understanding (Harel et al., 2019).

The modeling activity "Cycling" used in this study was designed on the STEP platform according to specific design principles (for further details, see Touma and Olsher, 2022). It includes four example-eliciting tasks that are designed so that the first task is an introductory task that helps students familiarize themselves with the content and tools, and the advanced tasks offer opportunities for higher cognitive demands. Some of the tasks include digital simulations and tools that allow interaction with the various components of the model, offloading part of the student's mathematical work to these tools (Touma and Olsher, 2022). This activity deals with constructing position-over-time graphs appropriate for real-life situations. In the first task, students were asked to use a given diagram to drag blue points and submit three different examples of graphs that represent a girl named Nora's ride. The ride must meet two requirements: it ends at 13:00 and ends at home. The GeoGebra-based applet (like the left window in figure 2) included a 4-segment interactive graph that can be manipulated by dragging the blue points. The left blue point (a fixed point) represents Nora's starting location, and the right blue point represents Nora's finishing location, both relative to home, while the x-axis represents the time relative to 8 am.

In this study, we focused on the second task of the activity (described below), which, unlike the first task, includes a simulation. The uniqueness of the simulation, which adds value to the use of the graph, is that it presents features of the bicycle's motion, such as the change in position relative to home, the change in speed, and the change in direction. These animation features can enhance students' understanding of the underlying mathematics associated with the task. Beyond the given simulation, we reinforced these features through verbal descriptions (including non-critical characteristics in addition to the requirements of the task) that appear in the post-submission report produced automatically according to the data the students chose to submit.

The task

The students were given the following instruction: *Yael rides a bicycle once a week. In each ride, she passes a total distance of 50 km in 4 hours. Use the diagram below, drag the blue points and submit three different examples of graphs representing Yael's ride.*

The task included a GeoGebra-based applet (Figure 2) divided into two windows. The left window includes a 3-segment interactive graph that can be constructed by dragging blue points. The left blue point represents Yael's starting location, and the right blue point represents Yael's finishing location, both relative to home, while the x-axis represents the time relative to 8 am. The right window includes a simulation of Yael's ride (overview of the track) corresponding to the graph on the left side of the applet, which is displayed by clicking the play button, and measurements of each simulation state (total distance, starting hour, and total time). Indeed, the applet includes multi-linked representations (MLR), which enable students to investigate various input data by reflecting their actions in a different representation and thus see their own ideas differently.

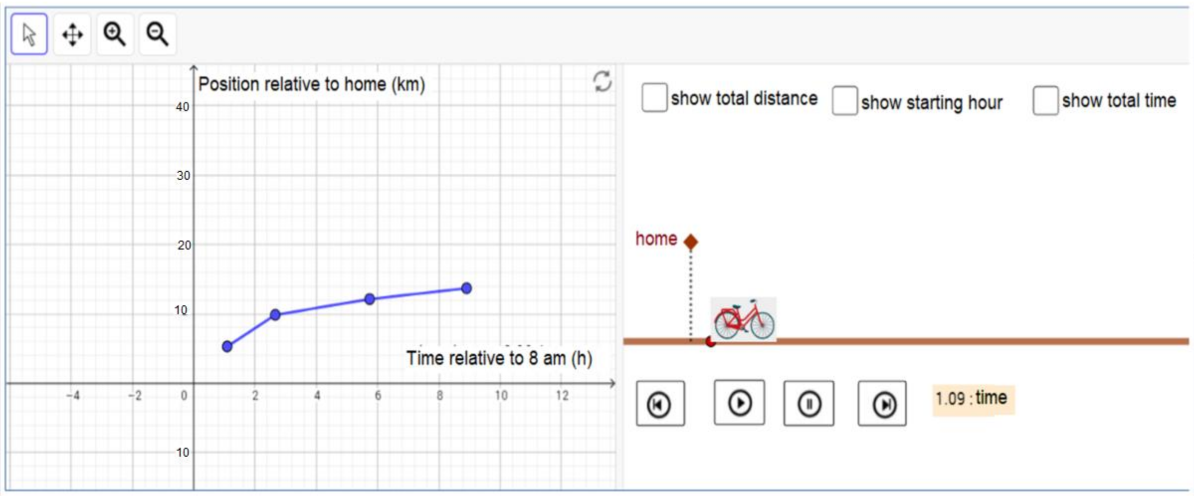

Figure 2: screenshot of the GeoGebra-based applet in task 2

Procedure

The pair of students worked together on the modeling activity within the mathematics class. They solved the tasks in the order in which they appeared in the activity. After completing the first task, the students did not refer to the post-submission report (despite the teacher's request to do so), but they did refer to it after completing the rest of the tasks.

Data collection and analysis

The data collected in the study consisted of (a) students' submissions from the STEP platform, (b) video recordings of the pair's computer screen while working on a modeling activity, and (c) field notes of the researcher taken during the pair observations.

To answer the first research question, the first author attended and observed the whole class, focusing on the pair of students while using the post-submission report. The recordings and notes taken during the observations were then transcribed. We analyzed all the statements and actions that the students performed during the interaction with the post-submission report, while focusing on which parts of the report they referred to. The first author defined and coded the data by mapping each statement and action within the MC and linking it to MMC. She then generated categories for students' statements and actions (detailed below). To answer the second research question, the first author examined the interactions she found while answering the first question and associated them with the modeling process. She formulated the answer to the second question by describing the routes within the MC.
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FINDINGS

Next, we present the submitted examples, the post-submission report of the second task (see Figures 3 and 4), and students' interactions with the post-submission report.

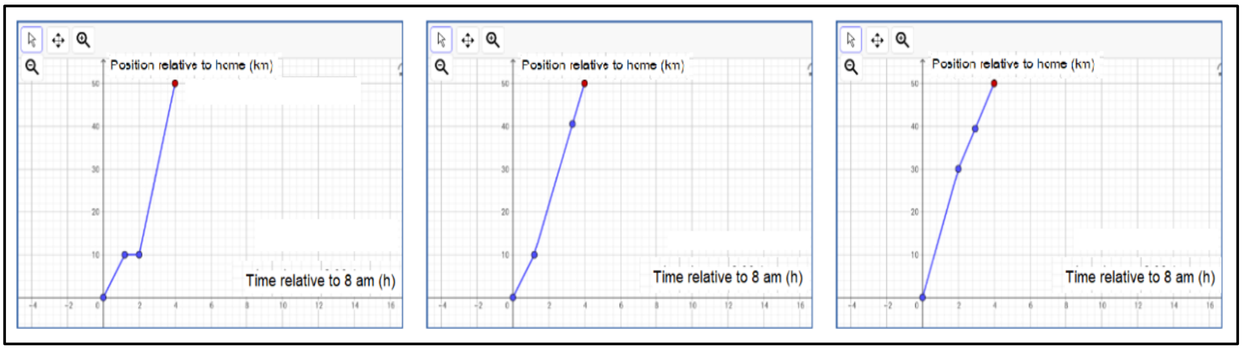

Figure 3: The three examples Ray and Lour submitted

After submitting three examples in the second task (represented in figure 3), Ray and Lour referred to the post-submission report. The post-submission report presented to the students (Figure 4) consisted of two lists that were prepared as part of the task design: (1) Task requirements: a list of critical characteristics that provide information on whether the example is an example of Yael's ride, as described in the task. Each characteristic is marked as V or X according to its existence in the example. (2) Example characteristics: A list of non-critical characteristics that can give students a sense of the breadth of their personal example space. This list includes the characteristics that the task designers would like to deepen the students' interaction with; therefore, the full list appears beneath each example. After analyzing the submitted examples, STEP highlights the characteristics that exist in each example in yellow. Among these characteristics are the various mathematical attributes of the different graphs and their implications on the real-life situation described in the task.

As shown in the report, the submitted examples met the task requirements of overall time and overall distance. In addition, the following eight characteristics were assessed in each example: Yael started the ride from home, Yael finished riding at home, Yael changed direction at least once, Yael stopped at least once, Yael rode at different speeds, Yael passed through home, Yael started the ride at 8 am, and Yael started the ride before 8 am. According to the automated analysis, three characteristics out of them existed in the three examples: Yael started the ride from home, Yael rode at different speeds, and Yael started the ride at 8 am.

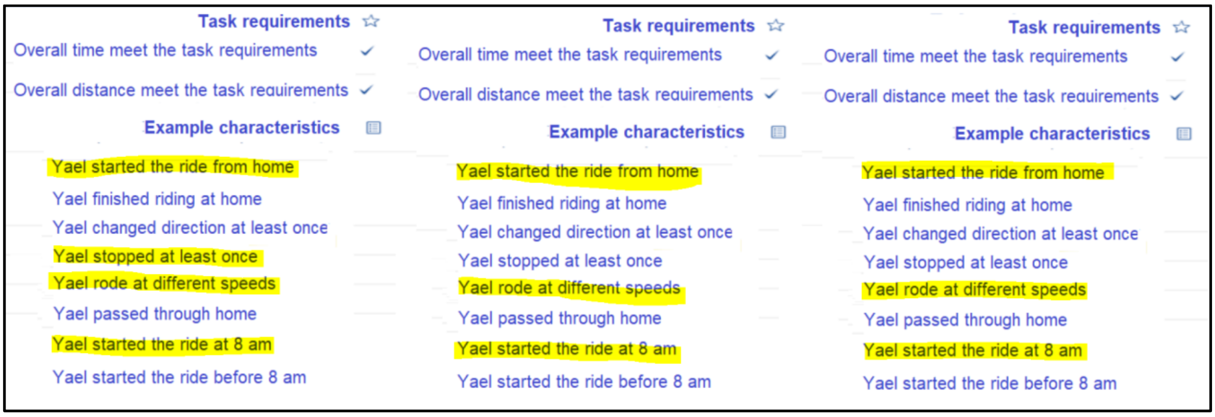

Figure 4: the post-submission report Ray and Lour received

Below are some of Ray and Lour's reactions to the post-submission report.

1 Lour: Yael started the ride from home, Yael stopped at least once, Yael rode at different speeds, Yael started the ride at 8 am. [Pointing at the characteristics highlighted in yellow in the list]

2 Ray: Yael finished riding at home. However, they did not tell us. [Pointing at the second characteristic on the list: "Yael finished riding at home," which appeared beneath all examples].

3 Ray: Yael changed direction at least once. [Pointing at the third characteristic in the list: "Yael changed direction at least once," which appeared beneath all examples].

4 Lour: Yael stopped at least once. [Pointing at the fourth characteristic in the list: "Yael stopped at least once," which appears beneath the left example].

5 Lour: Here she stopped. [Pointing at the graph of the left example]

6 Lour: Yael passed through home. Why was this not highlighted? [Pointing at the sixth characteristic in the list: "Yael passed through home," which appears beneath the three examples].

7 Ray: because we did not make her pass.

8 Ray: all answers are the same. We did a stop only in the left example. [Pointing at the fourth characteristic in the list: "Yael stopped at least once," which appears beneath the left example].

9 Ray: If she drove for 5 hours then this will be incorrect? [Asking the teacher while pointing at the first task requirement “overall time meet the task requirement”]

10 Teacher: right. You will get X beside the first task requirement

The following categories describe the students’ use of the post-submission report:

Validating correct answers

The validation process in mathematical modeling involves a series of critical steps aimed at ensuring that the obtained mathematical results accurately reflect real-world situations (Ferri, 2018). Contrary to the findings of Ramirez-Montes (2021) that the validation process was a step that students did not take, even though their answers were correct, in lines 1,4,5 and 8, the students used the verbal description within the report to validate the characteristics of their submitted examples. The interaction with the report enabled them to go through a control process in which the designed characteristics helped them reflect on their answers within the given situation and think about the differences between the graphs submitted. This is not what we would expect in the case of a "correct" answer and indicates the deepening of the interaction with the transition between a realistic situation and a mathematical situation.

The need of external validation

Lines 9-10 describe the interactions between Ray, the report, and the teacher. By asking the teacher about the effect of alternative input data on what is presented in the post-submission report, a “what if” situation, Ray ensures that she understood the first critical characteristic (overall time), and its mathematical representation. This interaction indicates that in some cases, external validation, in this case the teacher’s mediation, is needed in order to understand the results of the report, which is also attributed to the control step within the validation process.

Focusing on the task requirements

In line 2, Ray compared the results of the automated analysis with task requirements. She emphasized that the given situation did not require finishing the ride at home; therefore, they did not submit such an example of a graph. Thus, she validated the mathematical results in a given real situation. This might indicate that students prefer to focus on task requirements more than on other mathematical characteristics.

Unattended characteristics

Ray and Lour did not attend to the non-critical characteristic “Yael started the ride before 8 am” that appears at the end of the list. This might have happened because it was a negation of the characteristic before it, “Yael started the ride at 8 am.”

Next, we describe how students’ use of the post-submission report affects their MMC by describing the new nonlinear modeling route within the MC.

New non-linear modeling route

While associating the analyzed interactions (mentioned above) within the modeling process, we noticed that the interaction with the report, as part of the feedback process, enabled the students to perform a new route within the MC. The students, being at the "real results" phase, arrived to the "real situation" phase by proceeding within the MC in the following order: interpreting the real results presented as verbal descriptions in the post-submission report, validating the mathematical interpretations of the real situation with their submitted examples while using the mathematical model (the graphs), gaining insights to the real situation. For example, in lines 4-5, Lour noticed that the left example represents a situation in which Yael stopped at least once, although the trip description did not require that. Here, she performed a modeling route that included ensuring that the mathematical results obtained accurately reflected the real-world situation by emphasizing what appeared in the mathematical model (by pointing at the graph). Another example is shown in lines 6-7, by pointing at and reading the characteristic "Yael passed through home" Lour attended the real results presented as verbal descriptions. Indeed, this conversation validates the mathematical interpretations of a real situation with the submitted examples while using the mathematical model. In this situation, unlike Lour, Ray recognized that the mentioned characteristic did not exist in their examples, which shows that she made a connection between the mathematical model and the given verbal description. In both examples, the fact that the students made a connection with the given situation can be interpreted as a return to the real situation.

DISCUSSION AND CONCLUSIONS

The aim of the present study was to examine how an automated post-submission report can serve as a bridging tool between the literacy phenomenon and the given mathematical model. To this end, we explored students' use of post-submission reports while working on a modeling activity and its effect on their modeling process. Contrary to traditional mathematical problem solving, in which students often conclude their process with the acquisition of mathematical results, the validation process in mathematical modeling goes beyond (Ferri, 2018). The competencies required for effective validation involve critical checking and reflection on the solutions found. Technological tools can assist in this process not only by providing interactive simulations but also by enhancing the feedback process in other ways. Our findings illustrate this idea and answer the second research question: the verbal descriptions in the post-submission report helped Ray and Lour validate their answers by reviewing various parts of their solution within the given situation and the given mathematical model (graphs) by critically checking the submitted examples and reflecting on them. By interacting with the post-submission report, a non-linear modeling route was identified: from the real result (Phase 6) to the mathematical model (Phase 4) to the real situation (Phase 1). This new route enables students to interpret and validate mathematical results obtained in the extra-mathematical world. These findings support the idea that artifacts such as personal reports, which are designed as tools by others, may become instruments for learning (Yerushalmy et al., 2023) and contribute to the hypothesis that the post-submission report serves as a bridging tool between the literacy phenomenon and the given mathematical model.

Usually, reports are the last step in the feedback process and do not prompt further meaningful interaction with the task, especially if the answer is correct. In this study, the findings, which answer the first research question, show that the post-submission report is an integrative part of the ongoing feedback process, which enables students to make sense of information relating to the task and regarding aspects of their understanding (Hattie & Timperley, 2007). Indeed, looking at the whole interaction of the students with the task as a holistic feedback process that includes using the simulation and interpreting the results of the post-submission report contributes to the validation process that occurs while engaging with the report. For this purpose, a post-submission report is a pivotal part of the task. It should be noted that the findings for this study are preliminary, concerning one pair of students, but we will expand the sample to more pairs.

References

Abu-Raya, K., & Olsher, S. (2021). Learning analytics based formative assessment: Gaining insights through interactive dashboard components in mathematics teaching [Paper presentation]. *AI for Blended-Learning: Empowering Teachers in Real Classrooms Online Workshop*, *EC-TEL'21*, Bozen -Bolzano, Italy.

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, 1-32. https://doi.org/10.1007/s10649-021-10104-6

Ferri, R. B. (2018). *Learning how to teach mathematical modeling in school and teacher education. Springer*. https://doi.org/10.1007/978-3-319-68072-9

Greefrath, G. (2011). Using technologies: New possibilities of teaching and learning modelling—Overview. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), Trends in Teaching and Learning of Mathematical Modelling (ICTMA 14), (pp. 301–304). Springer. https://doi.org/10.1007/978-94-007-0910-2_30

Harel, R., Olsher, S., & Yerushalmy, M. (2022). Personal elaborated feedback design in support of students' conjecturing processes, *Research in Mathematics Education*, Advance online publication: https://doi.org/10.1080/14794802.2022.2137571

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of educational research, 77*(1), 81-112. https://doi.org/10.3102/003465430298487

Sadler, D. R. (1989). Formative assessment and the design of instructional systems. *Instructional science, 18*(2), 119-144

Sangwin, C. J., & Köcher, N. (2016). Automation of mathematics examinations. *Computers & Education, 94,* 215-227. https://doi.org/10.1016/j.compedu.2015.11.014

Touma, M., & Olsher, S. (2022). Designing computer-based modeling activities that support students' modeling process. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), Proceedings *of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12),* pp. 1167-1174.

Yerushalmy, M., Olsher, S., Harel, R., Chazan, D. (2023). Supporting inquiry learning: An intellectual mirror that describes what it "sees." *Digital Experiences in Mathematics Education. 9,* 315–342. https://doi.org/10.1007/s40751-022-00120-3